NATIONAL UNIVERSITY OF UZBEKISTAN
INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKY AND INSTITUTE OF NUCLEAR PHYSICS THE ACADEMY OF SCIENCES OF THE REPUBLIC OF UZBEKISTAN
STEKLOV MATHEMATICAL INSTITUTE OF RAS
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OF THE INTERNATIONAL SCIENTIFIC CONFERENCE OF “CONTEMPORARY MATHEMATICS AND ITS APPLICATION”

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CONTEMPORARY MATHEMATICS AND ITS APPLICATION

A B S T R A C T S

19-21 November 2021, Tashkent, Uzbekistan

Contemporary mathematics and its application: abstracts of the international scientific conference (19-21 November 2021, Tashkent, Uzbekistan). – Tashkent. 2021. p. 156

The book of abstracts contains the brief description of talks of the participants of the international conference "**Contemporary mathematics and its application**". The topics are related to mathematical modelling of nonlinear processes, algebra and functional analysis, differential equations and dynamical systems, ill-posed and inverse problems, mathematical analysis, geometry and topology, computational mathematics, statistical modelling.

This collection is intended for specialists in mathematics, applied mathematics and information technology, university teachers and for PhD, master students.

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INTERPRETATION OF HYPERBOLIC SPACES

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In geometry, stereometric mapping of elliptic spaces into Euclidean space is often used, which makes it possible to use the Cartesian coordinates of Euclidean space.

There is another method for mapping elliptic space to Euclidean space. This is a projection of a sphere representing an elliptical space from the center of the sphere to the tangent plane. In this case, diametrically opposite points are displayed in one point or a hemisphere can be considered displayed.

This method was used by A.V. Pogorelov, when considering problems of geometry "in the large" in elliptic space and in Lobachevsky space, and was called name Pogorelovsky mapping.

In [1], an analogue of the Pogorelovsky mapping was constructed for all elliptic and hyperbolic spaces with projective metrics.

The following properties of the Pogorelovsky mapping are proved.

Theorem. Under the Pogorelov mapping of semi-elliptic and semi-hyperbolic spaces.

$m-$planes are mapped to a plane.

Convex sets to convex.

The order of the metrics are preserved.

Semi-hyperbolic spaces are mapped onto the interior of a sphere of semi-euclidean space.

This mapping makes it possible to study problems of geometry "in the large" in semi-Euclidean and semi-hyperbolic spaces.

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ON REGULAR PARABOLICITY OF THE COMPLEMENTS OF WEIERSTRASS ALGEBROIDAL SETS

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In this work we use the next definitions (see [1,2]).

Definition 1. A Stein manifold X is called $S-$***parabolic*** if there exist special plurisubharmonic exhaustion function $\rho(z) \in psh(X)$ that is maximal outside a compact subset of X. If in addition we can choose $\rho(z)$ to be continuous then we will say that X is S^*-***parabolic***.

Let X is $S-$ parabolic manifold and $\rho(z)$ is special exhaustion function. We denote by $O(X)$ class of all analytic functions on X

Definition 2. If for a function $f(z) \in O(X)$ there exist positive numbers c and d such that for each $z \in X$ it holds inequality

$$\ln|f(z)| \le d\rho^+(z) + c, \quad (1)$$

where $\rho^+(z) = \max\{0,\ \rho(z)\}$, then the function $f(z)$ is called $\rho-$polynomial on X. Minimal integer value of d which satisfies (1) is called degree of the polynomial (as it shows examples in general minimal of such d may be noninteger).

For each $d > 0$ we denote by $\mathcal{P}_\rho^d(X)$ the set of all $\rho-$polynomials of degree less or equal to d and by $\mathcal{P}_\rho(X) = \bigcup\limits_{d=0}^{\infty} \mathcal{P}_\rho^d(X)-$ the set of all $\rho-$ polynomials on X. Class of polynomials on a parabolic manifolds may be very poor, even trivial, i.e. $\mathcal{P}_\rho^d(X) = \{const\}$. In this paper we are concenterated in the spacial class of prabolic manifolds which we call regular parabolic manifolds.

Definition 3. (A.Aytuna, A.Sadullaev [2]). $S-$parabolic manifold X is called ***regular*** in case if the space of all $\rho-$polynomials $\mathcal{P}_\rho(X)$ is dense in $\mathcal{O}(X)$. The main results of the paper are the next

Theorem 1. Let we are given a Weierstrass algebroidal set

$$A = \left\{z = (`z, z_n) \in \mathbb{C}^n : F(z) =: z_n^m + f_{m-1}(`z)z_n^{m-1} + ... + f_1(`z)z_n + f_0(`z) = 0\right\},$$

and coefficients $f_j('z), j = 0, 1, 2, ..., m-1$, are entire functions of the variables $'z = (z_1, z_2, ..., z_{n-1}) \in \mathbb{C}^{n-1}$. If at least one of the coefficients $f_j('z), j = 0, 1, 2, ..., m-1$, differs from usual polynomial in $'z$, then analytic function f on the S^*- parabolic manifold $X = \mathbb{C}^n \backslash A$ with the special exhaustion function $\rho(z) = \frac{1}{2}\ln\left(|`z|^2 + \left|F(z) + \frac{1}{F(z)}\right|^2\right)$, is $\rho-$ polynomial of a degree d if and only if this function admits next finite expansion

$$f(z) = \sum_{|k|=0}^{d} a_k {z_1}^{k_1} {z_2}^{k_2} ... z_{n-1}^{k_{n-1}} (F(z))^{k_n} + \sum_{|k|=0}^{d-1} b_k {z_1}^{k_1} {z_2}^{k_2} ... z_{n-1}^{k_{n-1}} \left(\frac{1}{F(z)}\right)^{k_n+1},$$

and here a_k and b_k are constant coefficients.

Theorem 2. Parabolic manifold $X = \mathbb{C}^n \backslash A$ is regular if and only if the function $f(z) = z_n$ can be approximated by $\rho-$ polynomials in X.

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BEHAVIOR OF HAUSDORFF DIMENSION OF CIRCLE MAPS WITH BREAKS

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The present work is devoted to the study of the monotonicity invariant measure of circle homeomorphisms breaks on the basis of the thermodynamic formalism. For the first time in the theory of dynamical systems, the thermodynamic formalism was introduced in the pioneering work of Ya.G. Sinai. Subsequently, thermodynamic formalism was developed in the works D. Ruelle, R. Bowen and others.

It is well known that every orientation-preserving circle homeomorphism T with irrational rotation number ρ is strictly ergodic, that is, it has a unique probability T -invariant measure $\mu = \mu_T$ and the invariant measure μ of smooth diffeomorphisms with typical irrational rotation number ρ is absolutely continuous w.r.t. Lebesgue measure ℓ on the circle [1]. A natural extension of circle diffeomorphisms are piecewise smooth homeomorphisms with break points. The regularity properties of invariant measures of piecewise smooth circle homeomorphisms are quite different from the case diffeomorphisms.

A. Dzhalilov and K. Khanin proved [2] that for a circle homeomorphism T from class $C^{2+\varepsilon}(S^1 \setminus \{x_b\})$, $\varepsilon > 0$, with one breakpoint x_b and irrational rotation number ρ_T the invariant measure μ_T is singular with respect to the Lebesgue measure ℓ, that is, there is a measurable subset $A \subset S^1$ such that $\mu_T(A) = 1$ and $\ell(A) = 0$.

One of the important characteristic of the singular invariant measure μ is the Hausdorff dimension $HD(\mu)$ (the exact bound on the dimensions of the sets of "full" measure μ). The well-known Frostman's lemma states that the Hausdorff dimension of $HD(\mu)$ coincides with $\underline{\tau}(\mu)$. Consider a homeomorphism of the circle $T \in C^{2+\varepsilon}(S^1 \setminus \{x_b\})$ with one break point $x_0 = x_b$ and jump ratio c, that is $\sqrt{\frac{T'_-(x_0)}{T'_+(x_0)}} = c \neq 1$. Denote by $I^{(n)}(x)$ the segment of the dynamical partition $\mathbb{P}_{m,n}(x_0)$ containing the point x. A. Dzhalilov and J. Karimov in [3] proved existence of the following limit:

$$\lim_{n\to\infty} \frac{\ln\left|I^{(n)}(x)\right|}{\ln \mu(I^n(x))} = \mu(c) \tag{1}$$

Now we formulate the next theorem.

Theorem. Let $T \in C^{2+\varepsilon}(S^1 \setminus \{x_b\})$, $\varepsilon > 0$ a circle homeomorphism with one break point x_b, irrational rotation number ρ and jump ratio c. Then $\mu(c)$ is increasing function on R^1.

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LORENTZ MANIFOLDS WHOSE RESTRICTED CONFORMAL GROUP HAS MAXIMAL DIMENSION

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The purpose of this work is to address the question of the explicit description and characterization of oriented, time-oriented, conformal Lorentz manifolds of dimension $n + 1 \geq 3$ whose restricted conformal group is of largest possible dimension. Up to some extent, conformal Lorentz geometry can be developed in analogy to the conformal Riemannian case. However, at a more substantial level, there are fundamental differences. For instance, it is well-known that a Riemannian manifold of dimension ≥ 3 whose conformal group has the largest possible dimension is the standard sphere with its natural conformal structure. In contrast to this Riemannian case, there are several examples of nonequivalent conformal Lorentz manifolds of dimension ≥ 3 whose conformal group is of maximal dimension. These include the conformal compactifications $\mathcal{E}^{1,n}_{\mathrm{I}}$ and $\mathcal{E}^{1,n}_{\mathrm{II}}$ of Minkowski $(n+1)$-space, realized respectively as the sets of all oriented and unoriented null lines through the origin of pseudo-Euclidean space $\mathbb{R}^{2,n+1}$ with the conformal Lorentz structures inherited from $\mathbb{R}^{2,n+1}$, and their universal covering $\mathcal{E}^{1,n} \cong \mathbb{R} \times S^n$ with the conformal structure induced by the product metric $-dt^2 + g_{S^n}$, the Einstein static universe. This can be seen as a manifestation of the failure in the non-Riemannian case of the rigidity theorem of Lelong-Ferrand, Obata and Alekseevsky, asserting that a Riemannian manifold $(\mathbb{M}, g)$ with a conformal group strictly larger than the isometry group of any metric in the conformal class of g is conformally equivalent to the standard sphere or the Euclidean space of the same dimension [1,2,3].

For an oriented, time-oriented, conformal Lorentz manifold (M, g) of dimension $n + 1 \geq 3$, we let $\mathrm{C}^{\uparrow}_{+}(M)$ denote the restricted conformal group of M, i.e. the group of conformal transformations preserving orientation and time-orientation.

We formulate our main results as follows:

Theorem 1. Let M be a $(n+1)$-dimensional $(n>1)$ Lorentzian oriented, time oriented conformal space, then its restricted Conformal group $\mathrm{C}_+^{\uparrow}(M)$ has a unique differentiable structure such that $\mathrm{C}(M)$ is a Lie-transformation group (i.e. $\mathrm{C}(M)$ is a Lie group and the action of $\mathrm{C}(M)$ on M is differentiable) such that $dim[\mathrm{C}(M)] \leq (n+3)(n+1)/2$.

Theorem 2. The conformal Lorentzian spaces $\mathcal{E}^{1,n}$, $\mathcal{E}_{\mathrm{I}}^{1,n}$ and $\mathcal{E}_{\mathrm{II}}^{1,n}$ are not conformally equivalent each to the other and their restricted conformal automorphism group is transitive and has maximal dimension is conformally equivalent either to $\mathcal{E}^{1,n}$ or to $\mathcal{E}_{\mathrm{I}}^{1,n}$ or else to $\mathcal{E}_{\mathrm{II}}^{1,n}$.

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AN ANALOGUE OF THE KOTELNIKOV-SHANNON THEOREM FOR A FIELD WITH BOUNDED SPECTRUM ON A SPECIAL SET

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Abstract: This article investigates and deduces the Kotelnikov-Shannon theorem for a field with bounded spectrum on a special set. We will also be interested in the rate of convergence and the possibility of approximating homogeneous random fields on a plane by such fields. The theorems obtained are extensions and analogs of some results in [1], [2].

In this article, we will consider an analogue of the Kotelnikov-Shannon theorem for a field with bounded spectrum on a special set. Here we also describe the conditions for the rate of convergence and the possibilities for approximating fields with a bounded spectrum on a special set. This theorem is a continuation and analogue of some results obtained by I. Kluvanek and Yu.K. Belyaev.

Let $\xi(t,s), (t,s) \in R^2$ be random in the wide sense homogeneous on the plane with zero mean. We will assume that $\xi(t,s), (t,s) \in R^2$ is a random field with bounded spectrum in the following sense: the spectral measure of the field is concentrated on the set $D = \{(\lambda_1, \lambda_2) \in R^2 : |\lambda_1| + |\lambda_2| < \widetilde{\omega}\}$. If the field has a spectral density, then in this case it is equal to zero outside the set D. Using the theorem of Kluvanek [1] and the lemma of Yu.K. Belyaev [2] one can prove the theorem:

Theorem 1. Let $\xi(t,s), (t,s) \in R^2$ be a homogeneous random field with bounded spectrum and its spectral measure of the field concentrated on the set D. Then, for any $\omega > \widetilde{\omega}$, the decomposition

$$\xi(t,s) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \xi(\frac{\pi}{\omega}(k+n), \frac{\pi}{\omega}(k-n)) \cdot \frac{\sin[\omega(\frac{(t+s)}{2} - \frac{k\pi}{\omega})]}{\omega(\frac{t+s}{2} - \frac{k\pi}{\omega})} \cdot \frac{\sin[\omega(\frac{(t-s)}{2} - \frac{n\pi}{\omega})]}{\omega(\frac{t-s}{2} - \frac{n\pi}{\omega})} \qquad (1)$$

where the series in equality (1) converges in the mean square.

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ON UNIFORM CONSISTENCY OF AN ESTIMATION OF DENSITY FUNCTION OF STATIONARY STRICTLY LINEARLY POSITIVELY DEPENDENT IN QUADRANT RANDOM VARIABLES

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Definition 1. A sequence $\{X_n,\ n \geq 1\}$ of random variables is said to be pairwise positively dependent in quadrant (PPQD), if for any $r_i, r_j \in R\ i \neq j$, the following inequality $P\left(X_i > r_i, X_j > r_j\right) \geq P\left(X_i > r_i\right) P\left(X_j > r_j\right)$ holds.

Definition 2. A sequence $\{X_n,\ n \geq 1\}$ of random variables is said to be linearly positively dependent in quadrant (LPQD), if for any disjoint $A, B \subset N$ and positive $r_i \in R$ the random variables $\sum\limits_{i \in A} r_i X_i$ and $\sum\limits_{i \in B} r_i X_i$ are PPQD.

Definition 3. A sequence $\{X_n,\ n \geq 1\}$ of random variables is said to be strictly linearly positively dependent in quadrant (SLPQD), if for each non decreasing function $g(x)$ the sequence $\{g(X_n),\ n \geq 1\}$ is LPQD.

For SLPQD random variables we use the following coefficient of dependence

$$r(k) = \sup_{\substack{x,y \in R \\ i \in N}} \left[P\left(X_i > x, X_{i+k} > y\right) - P\left(X_i > x\right) P\left(X_{i+k} > y\right)\right], \qquad k \in N.$$

Let $\{X_n,\ n \geq 1\}$ be a stationary sequence random variables with function in the density $f(x)$ which is in a closed interval $[a; b]$. Let K be a fixed density function and h_n is the sequence of non-negative real numbers. Nuclear estimations for density function and their derivatives of an order r are usually defined as follows:

$$f_n(x) = \frac{1}{nh_n} \sum_{j=1}^{n} K\left(\frac{x - X_j}{h_n}\right), \qquad f_n^{(r)}(x) = \frac{1}{nh_n} \sum_{j=1}^{n} K^{(r)}\left(\frac{x - X_j}{h_n}\right)$$

Let introduce the following assumptions:

(A1) for all $n \in N$, $\sum\limits_{j \geq n} r(j) = O\left(n^{-(m-2)/2}\right)$ holds for some $m > 2$.

(A2) $K(.)$ is a density function with bounded variation on R satisfying

(I) $\lim\limits_{|u| \to \infty} |u| K(u) = 0$; (II) $\int\limits_{-\infty}^{\infty} u^2 K(u)\, du < \infty$.

(A3) $K(u)$ is differentiable and $\int\limits_{-\infty}^{\infty} K'(u)\, du < \infty$.

(A4) for the integers $r \geq 1$ there exist bounded derivatives of order $r+1$ and

$$K^{(s)}(u) \to 0 \text{ as } u \to \infty, \quad s = 0, 1, ..., r-1.$$

(A5) $\{h_n\}$ and $\{m_n\}$ are sequences, such that, $h_n \to 0$, $m_n \to \infty$, $m_n h_n^{\nu+r+1} \to \infty$ as $n \to \infty$ and $\sum\limits_{n \geq 1} m_n n^{-m/2} h_n^{-(\nu+r+1)m} < \infty$ for some $\nu \geq 0$, $m > 2$.

Theorem. Let $\{X_n,\ n \geq 1\}$ be a stationary sequence of SLPQD random variables. We assume that conditions (A1) - (A5) hold with $\nu = 0$. Then as $n \to \infty$

$$\sup_{x \in I} \left| f_n^{(r)}(x) - f^{(r)}(x) \right| \to 0 \quad \text{a.s.}$$

THE SOLUTION OF NIELS BOHR'S PROBLEM ON INTERACTIVE COMMUNICATION

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One of the most urgent problems of our time is the security of information transfer. This can be seen even from the fact how much spam we receive every day by email. It is known that Advanced Encryption Standard, which is the basis of western information encryption, is based on such chaotic actions as permutation of cells, columns and matrix rows, which are the conversion of plaintext to ciphertext. These actions are random in nature and therefore do not provide complete confidentiality of information. Complete closeness of information can be provided if each information cell is closed using its own transformation. Such a complete set of transformations can be obtained by solving the equations for a function of N variables, where N is the number of cells. As it is known, there are very few exactly solvable equations for functions of N variables. One of the most reliable is the Lieb-Liniger model for describing the system of bosons interacting by means of delta-function potentials. This problem was first solved by Lieb and Liniger and is known in the scientific literature as the Lieb-Liniger model. Another vulnerable point leading to the loss of information security is the process of the encryption key transmitting after sending encrypted information from the sender (Alice) to the recipient (Bob). This vulnerability can be eliminated if Alice and Bob have their own encryption keys. Researchers drew attention to the problem of having their own encryption keys long before the development of modern information technologies.

Back in the early 30s of the twentieth century, an attempt to play poker at a distance between Professor Niels Bohr with his son, Heisenberg and other colleagues was unsuccessful, and a problem arose for the players to have their own encryption keys. Only in the 80s of the 20th century, Adi Shamir indicated a way to solve this problem. His method of solving the problem is often called a three-step protocol. It consists of the following steps. Alice encrypts the information with her encryption key and sends it to Bob. Bob encrypts the received information with his own encryption key and returns the information now under the two encryption keys back to Alice. Alice, having received this information, decrypts it with her decryption key and sends the information now under one encryption key back to Bob. Information is now under one encryption key with Bob. Bob, having received this information from Alice, decrypts it with his decryption key. Now the information is without an encryption key and Bob can get acquainted with the information that Alice wanted him to transfer. In this paper, we show the possibility of using expressions defined based on the Lieb-Liniger work as commutative Alice and Bob encryption keys for transmitting information based on a three-step protocol. It is shown that to determine the amount of time-dependent information, one can use the solution of the Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy of quantum kinetic equations, when the equilibrium density matrix is determined through the Bethe ansatz.

THE ANALOGUE OF BISHOP'S FORMULA IN A MATRIX BALL

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Let $Z = (Z_1, ..., Z_n)$ be a vector, where $Z_j, 1 \leq j \leq n$ is a square matrix of order m, over the field of complex numbers $\mathbb{C}$. We define a matrix scalar product for $Z, W \in \mathbb{C}^n[m \times m]$ in the following way ([1, p.12]):

$$\langle Z, W\rangle = Z_1 W_1^* + \cdots + Z_n W_n^*.$$

The following domain

$$B_{m,n} = \{Z \in \mathbb{C}^n[m \times m] : \ I - \langle Z, Z\rangle > 0\},$$

is called a *matrix ball*, where I is an identity matrix of order m.

It is known ([1, p.106]), that for any function $f \in H^1(B_{m,n})$ the following is true

$$f(Z) = \int_{X_{m,n}} \frac{f(W)d\sigma(W)}{det^{mn}(I - \langle Z, W\rangle)}, \quad Z \in B_{m,n}, \tag{1}$$

where $d\sigma(W)$ is a normalised Lebesgue measure on the skeleton $X_{m,n}$.

For a holomorphic mapping $f : G \to \mathbb{C}^n[m \times m]$ we consider the following matrix polyhedron set

$$f^{-1}\big(B_{m,n}\big) = \{Z \in G : \ rI - \langle f(Z), f(Z)\rangle > 0, \ r > 0\}.$$

The connected component of the matrix polyhedron set $f^{-1}\big(B_{m,n}\big)$ is called *a special matrix polyhedron* and we denote it by $\Theta_{f,r}$. The skeleton of the domain $\Theta_{f,r}$ is defined as follows:

$$\Gamma_{f,r} = \{Z \in G : \ \langle f(Z), f(Z)\rangle = rI, \ r > 0\}.$$

Let $f = (f_1, ..., f_n) : D \longrightarrow G$ be a holomorphic mapping of domains $D \subset \mathbb{C}^n_Z[m \times m], G \subset \mathbb{C}^n_W[m \times m]$. A mapping f is of finite type, if for each $W \in G$ the equation $f(Z) = W$ has the same finite number of roots in the domian D taking into account their multiplicities ([2, p.38]).

In this paper it is obtained Bishop integral formula in $\Theta_{f,r}$ by using formula (1) for a special form of functions $\frac{h(Z)}{J_f(Z)}$, where $h(Z) \in H^1(\Theta_{f,r})$, J_f is the Jacobian ($J_f(Z) \neq 0, Z \in \Theta_{f,r}$) of the map f of finite type.

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SPECIFIC FEATURES OF POLYTOP APPROXIMATION TO A SURFACE IN GALILEAN SPACE

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Development of methods A. D. Aleksandrov in the monograph "Intrinsic geometry of convex surfaces" makes it possible to study the intrinsic geometry of regular convex surfaces using the geometry of polyhedron. But the degeneracy of the metric of the Galilean space does not allow the direct application of these methods in the study of surfaces of the Galilean space. Since the distance, determined by the first part of the metric, between two points does not depend on the path connecting these points.

In [1], the definition of the shortest path on the surface is given as a curve on the surface with the smallest variation of the rotation. We have generalized the definition of a shortest path given in [1] for polyhedron in Galilean space ${}^1R_3$. We have proved the existence of a shortest path connecting two points of polyhedron with no special support planes.

The following theorem is also proved.

Theorem. If the sequence of polyhedron M_n tends to surface F, then "geodesics" at M_n tend to "geodesics" surfaces F.

A geodesic is understood as a continuous curve that is the shortest path on any sufficiently small segment.

In order to preserve the order of the metric, we require an approximation of the points of the polyhedral $M_n \to F$ with coordinates $(x_i, \ y, \ z) \in F$. To the coordinates $x_i = x_i^0 -$ retains the original values.

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Section 2: Mathematical and statistical modelling

MIXED CONVECTION AND JOULE HEATING EFFECTS ON MHD OLDROYD-B NANOFLUID FLOW OVER A STRETCHING SHEET THROUGH A POROUS MEDIUM IN PRESENCE OF NON-LINEAR THERMAL RADIATION, HEAT GENERATION/ABSORPTION, VISCOUS DISSIPATION AND CHEMICAL REACTION

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Keywords Oldroyd-B fluid, Nanofluid, Joule heating, Chemical reaction, Thermal radiation, Porous media, Heat generation.

Abstract. Study of non-Newtonian Oldroyd-B nanofluid flowover a stretching surface through a porous medium under the influences each of magnetic field, mixed convection, non-linear thermal radiation, heat generation/absorption, viscous dissipation and chemical reaction is the main objective of this research, taking into account the effects of Brownian motion coefficient and thermophoresis coefficient. On the other hand, the nonlinear partial differential equations governing the boundary layer flow have been transformed into a system of nonlinear ordinary differential equations using the similarity transformation and nondimensional variables, knowing that the new system of equations has been solved numerically by forth-order Runge-Kuttamethod with shooting technique, the results of the current study were interpreted by the graphs which illustrated the impacts of the all physical parameters on the distributions of velocity, temperature and concentration of nanoparticles using MATLAB program.Some results of the current study showed that the magnetic field parameter M and Darcy number Da negatively affect the velocity distribution $f'(\eta)$ in contrast to the effect of the mixed convection parameter δ_m on this distribution. Also, the effects of Lewis number Le, Prandtl number Pr and Brownian motion parameter Nb were negative on the distribution of concentration of nanoparticles $\phi(\eta)$. On the other side, the enhancement in the temperature distribution $\theta(\eta)$ was due to the effect of the nonlinear thermal radiation parameter R, Eckert number Ec, the heat generation and absorption parameter S, and the thermophoresis parameter Nt.

CONVECTION AND RADIATION MODE ON MHD FLOW OF JEFFREY NANOFLUID ON A STRETCHING SHEET THROUGH A POROUS MEDIUM IN PRESENCE OF JOULE HEATING, VISCOUS DISSIPATION,CHEMICAL REACTION WITH SLIP CONDITION

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Keywords Viscoelastic fluid, Jeffrey fluid, Nanofluid, Joule heating, Porous media, Chemical reaction, Thermal radiation.

Abstract. Study of slip and convection boundary conditions limits on magnetohydrodynamic Jeffery nanofluidflow on a stretching sheet through a porous medium under effects of nonlinear thermal radiation, Joule heating and viscous dissipation in presence of heat generation/absorption and chemical reaction is the focus of this research. On the other hand, thesystem of governing nonlinear partial differential equations which study the fluid flow process has been transformed into a system of nonlinear ordinary differential equations using similarity transformations and non-dimensional variables,which was later solved numerically by using fourth-order Runge-Kutta method with shooting technique. MATLAB program was used to make graphs that show the results of the influences of all physical parameters on the distributions of velocity, temperature and concentration of nanoparticles.It has been noted that some of the results of this study showed that the velocity distribution $f'(\eta)$ decreased under the influence of the magnetic field parameter M, Darcy number Da, Jeffrey fluid parameter λ and slip velocity parameter K_s, while this distribution increased under the influence of Deborah number β.On the other hand, effects of the non-linear thermal radiation parameter R, the ratio temperature parameter Ct, the Eckart number Ec, the Biot number Bi, and the heat source/sink parameterS had a positive effect on the temperature distribution $\theta(\eta)$ and also the concentration of nanoparticle distribution $\phi(\eta)$.

MATHEMATICAL MODELING IN THREE-DIMENSIONAL SOIL

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As you know, currently, by mathematical modeling of natural phenomena, opportunities for predicting expected catastrophes are being created. Taking this into account, in this work the propagation of radon in various environments directly related to nature was studied.

Until now, the distribution of radon in various media has been studied mainly taking into account advection and diffusion. However, theoretical estimates of these parameters obtained using classical models do not allow us to explain the abnormally high values of radon volume activity near the Earth's surface.

However, at the first stage of modeling, it is necessary, first of all, to have an idea about the propagation of radon in the soil-atmospheric system in time and space. To do this, it is necessary to develop a model of unsteady radon transport in the soil - atmospheric system and obtain its analytical solution.

To do this, consider a mathematical model described by the following system of equations:

$$\begin{array}{l}
\frac{\partial \mathrm{A}}{\partial t} = D_a \frac{\partial^2 \mathrm{A}(z,t)}{\partial z^2} + v_a \frac{\partial \mathrm{A}(z,t)}{\partial z} - \lambda \mathrm{A}\left(z,t\right), z<0,\\
D_g \frac{\partial \mathrm{A}(z,t)}{\partial z} + v_g \frac{\partial \mathrm{A}(z,t)}{\partial z} - \lambda\left(\mathrm{A}\left(z,t\right) - \mathrm{A}_\infty\right), z>0,\\
D_g \left.\frac{\partial \mathrm{A}(z,t)}{\partial z}\right|_{z=0+0} + v_g \left.\mathrm{A}\left(z,t\right)\right|_{z=0+0} = D_a \left.\frac{\partial \mathrm{A}(z,t)}{\partial z}\right|_{z=0-0} + v_a \left.\mathrm{A}\left(z,t\right)\right|_{z=0-0},\\
\left.\mathrm{A}\left(z,t\right)\right|_{z=0+0} = \left.\mathrm{A}\left(z,t\right)\right|_{z=0-0}, \ \lim\limits_{z\to\infty} \mathrm{A}\left(z,t\right) = \mathrm{A}_\infty, \ \lim\limits_{z\to-\infty} \mathrm{A}\left(z,t\right) = 0,
\end{array}$$

In conclusion, we can say that the prediction of natural phenomena using the aforementioned mathematical modeling is achieved in a scientific way in order to prevent natural disasters as much as possible.

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GLOBAL SOLVABILITY AND EXPLICIT ESTIMATION OF SOLUTIONS OF A CROSS-DIFFUSION PARABOLIC SYSTEM IN NON-DIVERGENT FORM WITH A SOURCE AND VARIABLE DENSITY

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The development of science and technology requires the early prediction of processes in nature and their rational use. Many processes in nature are represented by equations and systems of equations of the parabolic type using the Cauchy problem. Many scientists have conducted their own research on this type of parabolic equations and systems of equations [1-2]. In particular, there is a growing demand for solving Cauchy problems and studying its properties in non-divergent parabolic type systems [3-5]. In this work, we consider in $Q = \left\{(t,x): \ t>0, \ x \in R^N\right\}$ parabolic system of nonlinear equations not in divergence form with variable density

$$|x|^n \frac{\partial u_i}{\partial t} = u_i^{\alpha_i} \nabla \left(u_{3-i}^{m_i-1} \left|\nabla u_i^k\right|^{p-2} \nabla u_i \right) + |x|^n u_i^{\beta_i} \tag{1}$$

$$u_i\left(0,x\right) = u_{0i}\left(x\right), x \in R^N \tag{2}$$

where $n, k, \ p, \ m_i \ , \beta_i, \alpha_i \ (i=1,2)$ the numerical parameters, $\nabla(\cdot) = grad_x\left(\cdot\right)$,

$u_i = u_i\left(x,t\right) \geq 0$ are the solutions. It is clear that the system (1) is degenerate. Therefore, it does not have classical solutions on the domain defined by equations. $u_i\left(t,x\right) = 0, \nabla u_i\left(t,x\right) = 0$ meaning system (1) may not have a classical solution. Therefore, in this case we consider a weak solution having the property $u_i^{\alpha_i} \nabla \left(u_{3-i}^{m_i-1} \left|\nabla u_i^k\right|^{p-2} \nabla u_i \right) \in C\left(Q\right) \ (i=1,2)$. Different particular cases of the problem (1)-(2) considered in many works (for instance see [1-5] and references therein). The system (1) describes a set of physical processes, for example process of mutual reaction-diffusions, heat conductivity, a polytropical filtration of a liquid and gas in the nonlinear environment whose capacity equal ${u_i}^{\beta_i}$.

Introduce the following notations

$a>0, \ A_i^{\alpha_i+k(p-2)} A_{3-i}^{m_i-1} = \frac{\frac{\gamma\gamma_i}{p+n} + \frac{\psi_i}{1-\beta_i}}{\gamma\gamma_i|\gamma\gamma_i|^{p-2}(\gamma b_i - N - n)}, \ \psi_i = \left(\frac{k(p-2)+\alpha_i}{1-\beta_i} + \frac{m_i-1}{1-\beta_{3-i}} + 1\right)^{-1}.$

$b_i = \left(\gamma_i k - 1\right)\left(p-2\right) + \gamma_{3-i}\left(m_i - 1\right) + \gamma_i - 1, \ T>0, \sigma = \frac{k(p-2)+\alpha_1}{1-\beta_1} + \frac{m_1-1}{1-\beta_2} + 1,$

$\gamma_i = \frac{(p-1)(k(p-2)+\alpha_{3-i}-m_i+1)}{k(p-2)(k(p-2)+\alpha_i+\alpha_{3-i})+\alpha_i\alpha_{3-i}-(m_i-1)(m_{3-i}-1)}, \ \gamma = \frac{p+n}{p-1}$

The following theorem are proved.

Theorem. (A global solvability). Let the conditions of $\gamma_i > 0$

$$\gamma b_i A_i^{\alpha_i+k(p-2)} \cdot A_{3-i}^{m_i-1} k^{p-2} |\gamma\gamma_i|^{p-2} = \frac{1}{p+n}$$

$$\psi_i\left(A_i^{\beta_i-1}a^{\gamma_i\beta_i-\gamma_i}-\frac{1}{1-\beta_i}\right)-\frac{(N+n)\,\gamma_i}{(p+n)\,b_i}\le 0$$

$$u\,(t,0)\le u_i(t,0)_+,x\in R^N$$

Then, for sufficiently small $u_{0\,i}\,(x)$ the followings holds:

$$u\,(t,x)\le u_i(t,x)_+=(T+t)^{\frac{1}{1-\beta_i}}\cdot A_i\left(a-\left(\frac{\sigma\cdot|r|}{(T+t)^{\frac{\sigma}{p+n}}}\right)^{\gamma}\right)_+^{\gamma_i}$$

in Q

The results obtained in the above theorem are a generalization of the results in [3]. If we take $k=1$ and $m_1=m_2=1$, in our given theorem, then the results in [3] are obtained.

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CROSS-DIFFUSION PARABOLIC SYSTEM EQUATIONS IN NON-DIVERGENT FORM IN NON-HOMOGENEOUS MEDIUM WITH TIME DEPENDENCY

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Consider the following cross-diffusive system of equations

$$\begin{cases}\dfrac{\partial u}{\partial t}=u^{l_1}\nabla\left(u^{\sigma_1}\nabla u\right)+t^{n_1}|x|^{m_1}u^{p_1}v^{q_1}\\ \dfrac{\partial v}{\partial t}=v^{l_2}\nabla\left(v^{\sigma_2}\nabla v\right)+t^{n_2}|x|^{m_2}u^{p_2}v^{q_2}\end{cases}$$

in $\Omega=\left\{(t,x):t\in R_+,x\in R^N,N\ge 1\right\}$ with Cauchy conditions

$$u|_{t=0}=u_0(x)\ge 0,\quad v|_{t=0}=v_0(x)\ge 0$$

where $l_i\ge 0,\sigma_i,n_i,m_i,p_i,q_i\ (i=1,2)$ are numerical parameters.
We seek the solutions $u(t,x),v(t,x)$ in the form:

$$u(t,x)=t^{-\alpha_1}f_1(\xi),\quad v(t,x)=t^{-\alpha_2}f_2(\xi)$$

where $\alpha_i=\dfrac{(m_i+2n_i+2)(l_{3-i}+\sigma_{3-i}-1)}{\Delta_\alpha}$, $(i=1,2)$

$\beta=\dfrac{q_1(l_1+\sigma_1-1)+(l_2+\sigma_2-1)(p_1-1-(n_1+1)(l_1+\sigma_1-1))}{\Delta_\alpha}$,

$\Delta_\alpha=2q_1(l_1+\sigma_1-1)+(l_2+\sigma_2-1)(2(p_1-1)+m_1(l_1+\sigma_1-1))\ne 0.$

If the condition $m_1 - m_2 = 2(n_2 - n_1)$ holds, then the previous system becomes the following a self-similar system of equations:

$$\begin{cases} \xi^{1-N} f_1^{l_1} \dfrac{d}{d\xi}\left(\xi^{N-1} f_1^{\sigma_1} \dfrac{df_1}{d\xi}\right) + \beta\xi \dfrac{df_1}{d\xi} + \xi^{m_1} f_1^{p_1} f_2^{q_1} + \alpha_1 f_1 = 0 \\ \xi^{1-N} f_2^{l_2} \dfrac{d}{d\xi}\left(\xi^{N-1} f_2^{\sigma_2} \dfrac{df_2}{d\xi}\right) + \beta\xi \dfrac{df_2}{d\xi} + \xi^{m_2} f_1^{p_2} f_2^{q_2} + \alpha_2 f_2 = 0 \end{cases}$$

We seek the f_i in the following form: $f_i(\xi) = A_i \xi^{\gamma_i}$, $(i = 1, 2)$
where $\gamma_i = \dfrac{(m_i + 2)(l_{3-i} + \sigma_{3-i} + 1) + (m_1 + 2)(i(p_2 + q_2) - p_2) - (m_2 + 2)(i(p_1 + q_1) - p_1)}{\Delta_\gamma}$,
$\Delta_\gamma = (l_1 + \sigma_1 + 1 - p_1)(l_2 + \sigma_2 + 1 - p_2) - p_2 q_1$
If inequality $\gamma_i(q_i + \sigma_i) > 2$ holds, then the last system has an asymptotic approximation solution.

Let, $\varepsilon_i = sgn(A_i(\alpha_i + \beta\gamma_i))$ $(i = 1, 2)$. Then f_i becomes:
- a sup(sub) solution, at ε_i = -1 ($\varepsilon_i = 1$);
- an analytical solution at $\varepsilon_i = 0, (i = 1, 2)$.

Results of a numerical experiments showed, usage estimates of solutions are good appropriate for construction of iteration process quickly converging to solution of the considered problem keeping nonlinear effects. We use as initial approximation the solutions of the self-similar equations constructed by the method of nonlinear splitting[1].

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FUJITA TYPE GLOBAL SOLVABILITY AND ASYMPTOTIC SELF-SIMILAR SOLUTION WITH DOUBLE NONLINEARITY

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In this talk we consider the following problem Cauchy

$$\begin{aligned} \tfrac{\partial u}{\partial t} &= \nabla\left(|x|^n u^{m_1-1} \left|\nabla u^k\right|^{p-2} \nabla u\right) + \varepsilon u^{p_1} v^{q_1}, \quad \varepsilon = \pm 1 \\ \tfrac{\partial v}{\partial t} &= \nabla\left(|x|^n v^{m_2-1} \left|\nabla v^k\right|^{p-2} \nabla v\right) + \varepsilon u^{p_2} v^{q_2} \end{aligned} \qquad (1)$$

$$u(0, x) = u_0(x) \geq 0, \quad v(0, x) = v_0(x) \geq 0, \quad \in R^N \qquad (2)$$

where $\varepsilon = \pm 1$, $m_1, m_2 > 1$, , $p_i, q_i \geq 1$, $(i = 1, 2)$ are positive real numbers. The numerical parameter n characterizes the variable permeability of a nonlinear medium.

Mathematical model (1) used for describe different nonlinear processes of heat transfer, diffusion, polytropic filtration in a nonlinear two-componential medium with variable permeability in the presence of absorption ($\varepsilon = -1$) and source ($\varepsilon = +1$) .equation (1) in the particular value of the numerical parameter (p=2) describes the process in porous medium. The functions u and v may be considered as the temperatures of two interacting with each other components of some fuel mixture [1]. System (1) with $u = 0, v = 0$ degenerates to the first-order system equation. In the domain of degeneration the problem (1),(2) may not have the classic solution. That is why we discuss the generalized solution having the property of continuity of solution and stream.

In this paper, we prove condition of the global solvability of the Fujita type to the Cauchy problem for nonlinear degenerate parabolic system (1) for slowly, fast diffusion, Fujita type critical and double critical cases. Estimate of the solution, and the front perturbation to the Cauchy problem depending on the value of the numerical parameters, asymptotic behavior of the self-similar solution with compact support and quenching solution discussed. Proofs of the proposals are based on the principle of comparison solutions. For establishing asymptotic of self-similar solution, the method The Cauchy problem and boundary value problems for the system of equations (1) in one-dimensional and multidimensional cases with different initial and boundary conditions studied by many authors [1-4]. of standard equation [2] suggested. Numerical results presented in visualized forms.

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ON THE DYNAMICS OF THE LOTKA-VOLTERRA COMPETITIVE-DIFFUSION SYSTEM

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Competition for natural resources regulates the growth of biological populations and leads to limited population growth. Moreover, two species competing for the same limiting resource often cannot coexist. The Lotka-Volterra model is used to describe competition for resources and predicts that elimination of competition will occur in the event of weak competition [1,2]. On the other hand, the spatial heterogeneity of the environment can change the outcome of competition, and the dynamic behavior of spatially explicit mathematical models can, to a certain extent, explain the ecological complexity of ecosystems. One of the main mathematical models for describing competition for resources in a spatially homogeneous environment is the following diffuse Lotka - Volterra competition system:

$$\begin{cases} d_1(u)u_t = \Delta u + u(a_1 - b_1 u - c_1 v) \qquad (x,t) \in \Omega \times (0,\infty); \\ d_2(v)v_t = \Delta v + v(a_2 - b_2 u - c_2 v) \qquad (x,t) \in \Omega \times (0,\infty); \end{cases} \tag{1}$$

$$\frac{\partial u}{\partial v} = \frac{\partial v}{\partial \nu} = 0 \quad (x,t) \in \partial\Omega \times (0,\infty); \quad u(x,0) = u_0(x); \quad v(x,0) = v_0(x) \times \partial\Omega;$$

where $\Omega \subset R^n$, $u(x,t)$ and $v(x,t)$ - population density of two competing species with diffusion rates $d_1(u)$ and $d_2(v) > 0$, parameters a_1, a_2 - their own growth rates, b_1, c_2 - coefficients of intraspecific competition, b_2, c_1- coefficients of interspecies competition. Suppose for now that $a_i, b_i, c_i, i = 1, 2$ are positive constants, i.e. the spatial environment is homogeneous. If the coefficients satisfy the condition of weak competition

$$\frac{b_1}{b_2} > \frac{a_1}{a_2} > \frac{c_1}{c_2} \tag{2}$$

then the corresponding elliptic system admits a unique positive steady state (u^*, v^*), given by the formula(u^*, v^*),

$$u^* = \frac{a_1c_2 - a_2c_1}{b_1c_2 - b_2c_1}; \qquad v^* = \frac{a_2b_1 - a_1b_2}{b_1c_2 - b_2c_1}; \tag{3}$$

Theorem 1. Suppose that the medium is homogeneous and (3) is satisfied. Then the positive steady state (u^*, v^*), is globally asymptotically stable among all nonnegative and nontrivial initial conditions.

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NUMERICAL SIMULATION OF SOLUTIONS OF DIFFUSION EQUATIONS IN PROBLEMS OF MULTI ASSET DERIVATIVES PRICING

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In this paper we consider the pricing of the multi-asset options by Monte Carlo method simulating of the solutions of diffusion equations. Options are one of the most important financial instruments for risk management and option pricing is a well known problem in financial mathematics. Pricing multi-asset options is amongst the most important and challenging problems in the modern financial industry. One of the most important of multi-asset options is basket option. Basket option , as a new type of multi-asset options, is often used to hedge a basket of assets, whose profit is determined by the weighted arithmetic average price of the basket assets. Essentially, it is a portfolio of multiple assets, which is usually cheaper than the price of a single asset option. Hence, a basket option is more efficient than portfolio of single option in view of cost in a basket. The option values of European, Asian, American and perpetual types are calibrated when given various payoffs and up to three assets.

We show how to reformulate the multi-asset option pricing by partial differential equation (PDE) with boundary value problems. Kolmogorov PDEs with affine-linear coefficient functions regularly appear in applications; the heat equation from physics and the classical and generalized Black-Scholes equations from computational finance are important examples of Kolmogorov PDEs with affine-linear coefficient for Asian basket options. Kolmogorov PDEs with boundary value problems are considered in order estimate the price of Asian basket options.We have at our disposal stochastic processes which solve the so-called backward stochastic differential equations. These processes provide us with a Feynman-Kac representation for the solutions of the Kolmogorov PDEs with boundary value problems.

We design a novel algorithm based on Monte-Carlo in order to approximate the solution of the backward stochastic differential equations (BSDEs). Our algorithm allows massive parallelization of the computations on many core processors such as graphics processing units (GPUs). Our approach consists of a novel method of stratification which appears to be crucial for large scale parallelization. In this way, we minimize the exposure to the memory requirements due to the storage of simulations. For the European and Asian basket options numerical results are given. The accuracy of our scheme is demonstrated on the test problems.

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DATA ENVELOPMENT ANALYSIS (DEA) MODELS FOR INNOVATION EFFICIENCY

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DEA was first developed by Farrel in 1957, which later been modified by Charnes-Cooper-and Rhodes (CCR) in 1978. It is a non-parametric method that utilizes linear programming to measure the level of efficiency of comparable decision-making units (DMU) by employing multiple inputs and outputs.[1]

$$Efficiency = \frac{f(Outputs)}{g(Inputs)}$$

where $f(.), g(.)$-some functions.

DEA can help to answer for managers:[2]

- How do I select appropriate role models to serve as possible benchmarks for a program of performance improvement?
- Which production facilities are the most efficient in my organisation?
- If all my operations were to perform according to best practice, how many more service outputs could I produce and how much could I reduce my resource inputs by, and in what areas?
- What are the characteristics of efficient operating facilities and how can they guide me in choosing locations for expansion?
- What is the optimum scale for my operations and how much would I save if all my facilities were the optimum size?
- How do I account for differences in external circumstances in evaluating the performance of individual operating facilities?

DEA analysis is used in various situations and sectors of economy to evaluate the organizational efficiency. So, it was used many wide range of input and outputs in the previous researches of the authors.

In this paper we consider the level of efficiency of comparable decision-making units (DMU) by employing multiple inputs and outputs for national innovation system. The inputs are: RandD expenditure,number of researcher,FDI. The output are: number of published papers, number of patients,GDP. Numerical results are obtained.

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ON PERIODIC SOLUTIONS TO SYSTEMS OF COMPETING SPECIES

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We consider strictly positive solutions of systems of competing species. It is known that in the presence of strictly positive solutions, new results on non-uniqueness and a number of other results are obtained that show how complex these systems can be. The approach to the problem is based on the method of upper and lower solutions and related monotonic iterations. This method leads to the existence of maximum and minimum periodic solutions, which can be calculated from a linear iterative process in the same way as for parabolic initial-boundary value problems.

Assuming that the resource function in spatial variables is decreasing, [1] described the competition between two aquatic organisms with different diffusion strategies for the same resource in the Lotka-Volterra reaction-diffusion-advection system. [2] studied the dynamics of competition between two organisms from the point of view of river ecology. An interesting feature of their system was that the boundary conditions at the upper and lower ends could represent a net loss of individuals. In some cases, both organisms leave the place of competition, neither coexist, nor die out.

Let us investigate the problem of coexistence and stability of the competition-diffusion model in population dynamics, which is presented in the form

$$u_t - d_1 u_{xx} = u(a_1 - b_1 u - c_1 v), \quad in \quad Q \tag{1}$$

$$v_t - d_2 v_{xx} = v(a_2 - b_2 u - c_2 v) \quad in \quad Q \tag{2}$$

$$u(t,-l) = u(t,l) \quad u_x(t,-l) = u_x(t,l) \quad t > 0 \tag{3}$$

$$v(t,-l) = v(t,l) \quad v_x(t,-l) = v_x(t,l) \quad t > 0 \tag{4}$$

$$u(0,x) = u_0(x) \quad -l \le x \le l \tag{5}$$

$$v(0,x) = v_0(x) \quad -l \le x \le l \tag{6}$$

where u and v are the densities of two competing species in a restricted habitat in $Q = \{(t,x) : t > 0, \mid x \mid < l\}$. The physical parameters d_i, a_i, b_i, c_i, $i = 1, 2$, and the boundary-initial functions u_0, v_0 are non-negative. It is shown from an established general condition that the trivial solution $(0,0)$ is always unstable; among the three nontrivial solutions only one is asymptotically stable while the remaining two are unstable. The stable one and the unstable ones are determined by the relative magnitude among the three constants a_1/a_2, b_1/b_2, c_1/c_2 independent of the diffusion coefficients d_i and the domain Q. The same criteria also determine whether the two competing species coexist or one wipes out the other. It turns out that these two species coexists with (η_1, η_2) as the asymptotic limit when $c_1/c_2 < a_1/a_2 < b_1/b_2$ and otherwise when either $a_1/a_2 < c_1/c_2$ or $a_1/a_2 > b_1/b_2$. The aim of the work is to prove some comparison theorems and study the problems of coexistence and stability of the competition-diffusion model.

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ASYMPTOTIC SOLUTION OF THE DOUBLE NONLINEAR PARABOLIC EQUATION WITH DAMPING IN SECONDARY CRITICAL CASE

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This paper discusses the asymptotic solution and global solvability solution of the Cauchy problem to the double nonlinear parabolic equation with damping for diffusion equation with a damping term as follows

$$u_t = div(u^{l-1}\left|\nabla u^k\right|^{p-2}\nabla u) - g(x,t)|\nabla u^m|^{p_1}u^{q_1},$$
$$u(0,x) = u_0(x),\ x \in R^N \tag{1}$$

where $l, m,\ k \geq 1,\ p \geq 2, p_1, q_1 \geq 0$ are the given numerical parameters, characterizing nonlinear media, $\nabla(.) = grad_x(.)$.

Different qualitative properties of the problem (1) in particular value of numerical parameters intensively studied by many authors (see [1-3] and references therein).

In this work we consider the case 1) $g(x,t) = g(x), N = 1$, 2) case $g(x,t) = t^\sigma, N \geq 1$. For the case 2) by analysis of following self-similar solution

$$u(x,t) = (T+t)^{-\alpha} f(\xi),\ \ \xi = |x|\,(T+t)^{-\beta},\ T \geq 0,\ \alpha, \beta > 0,$$

the Cauchy problem (1) the following condition of global solvability Fujita type i.e. $lp_1 + q_1 \geq (k(p-2)+m) + \frac{(\sigma+1)p-p_1}{N}$, an estimate of weak solution, the phenomenon of a finite speed of perturbation, an asymptotic self-similar solution of the equation (1) in the slowly and fast diffusion cases established. For the case 2) considered stationary case of the equation (1) which in literature [1] is known as generalized Emden- Fowler type equation

$$\frac{d}{dx}(u^{l-1}\left|\frac{du^k}{dx}\right|^{p-2}\frac{du}{dx}) - g(x)\left|\frac{du^m}{dx}\right|^{p_1}u^{q_1} = 0. \tag{2}$$

The asymptotic regular, finite and unbounded solutions via WKB solution of this equation (2) in the form $u(x) = f(x)z(\varphi(x))$, where the functions $f(x)$, $\varphi(x)$ are solution to the following system

$$f^{m+k(p-2)-q_1}(\frac{d\varphi}{dx})^{p-p_1} = g(x),$$

$$2\frac{df}{dx}\frac{d\varphi}{dx} + f\frac{d^2\varphi}{d^2x} = 0$$

and $z(\varphi(x))$ is the solution of equation

$$\frac{d}{d\varphi}(z^{l-1}\left|\frac{dz^k}{d\varphi}\right|^{p-2}\frac{dz}{d\varphi}) - \left|\frac{dz^m}{d\varphi}\right|^{p_1}z^{q_1} = 0$$

established.

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ON ONE LINEAR INVERSE PROBLEM FOR THREE-DIMENSIONAL TRIKOMI EQUATION WITH SEMI-PERIODIC BOUNDARY CONDITION IN AN UNBOUNDED PARALLELEPIPED

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In the domain
$G = Q \times \mathbb{R} = (-1, 1) \times (0, T) \times \mathbb{R}$ we consider the three-dimensional Tricomi equation:

$$Lu = x\, u_{tt} - \Delta\, u + a(x,t)\, u_t + c(x,t)\, u = \psi(x,t,z), \tag{1}$$

where $\Delta u = u_{xx} + u_{zz}$ is the Laplace operator, $\psi(x,t,y) = g(x,t,y) + h(x,t)\, f(x,t,y)$, the functions $g(x,t,y)$ and $f(x,t,y)$ are given and the function $h(x,t)$ is unknown.

Linear inverse problem. Find the pair of functions $\{u(x,t,z),\ h(x,t)\}$ satisfying the equation (1) in the domain G, the following semi-nonlocal boundary conditions

$$D_t^p\, u|_{t=0} = D_t^p\, u|_{t=T}\,, \tag{2}$$

$$u|_{x=-1} = u|_{x=1} = 0, \tag{3}$$

and the additional condition

$$u(x,t,\ell_0) = \varphi_0\,(x,t), \tag{4}$$

where $p \in \{0,1\},\ D_t^p u = \frac{\partial^p u}{\partial\, t^p},\ \ D_t^0\, u = u,\ \ell_0 \in \mathbb{R}$ with the function $h(x,t)$ belongs to the class

$$U = \Big\{(u,h)\,|\, u \in W_2^{2,s}(G);\ h \in W_2^2(Q),\ s \geq 3\Big\}.$$

Now, by the aid of the Fourier transform, we determine the space $W_2^{l,s}(G)$ with the norm

$$\|u\|^2_{W_2^{l,s}(Q)} = (2\pi)^{-1/2} \int\limits_{-\infty}^{+\infty} \left(1+|\lambda|^2\right)^s\, \|\hat{u}\,(x,t,\lambda)\|^2_{W_2^l(Q)}\, d\lambda, \tag{A}$$

where s, l are any finite positive integers. The Sobolev space is defined by $W_2^l(Q)$ (for $l = 0,\ W_2^0(Q) = L_2(Q)$) with the scalar product $(u, \vartheta)_l$ and with the norm

$$\|\vartheta\|_l^2 = \|\vartheta\|^2_{W_2^l(Q)} = \sum_{|\alpha|\leq l} \int\limits_Q |D^\alpha \vartheta|^2\, dxdt,$$

where α is multi-index, D^α is generalized derivative on variables x and t. It is obvious that the space $W_2^{l,s}(G)$ with the norm (A) is a Hilbert space. We need to introduce definitions of several function spaces and designations. The Fourier transform of function $u(x,t,z)$ we denote by

$$\hat{u}(x,t,\lambda) = (2\pi)^{-1/2} \int\limits_{-\infty}^{+\infty} u(x,t,z)\, e^{-i\lambda z} dz$$

Remark. The result is valid for the multidimensional Tricomi equation.

MONTE CARLO METHOD FOR COMPUTING MULTI-DIMENSIONAL INTEGRALS IN FINANCIAL PROBLEMS

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In financial mathematics, there are various types of options, the main ones include: American, European, Asian options. Methods that are used in financial mathematics to evaluate options are divided into analytical and numerical. Using analytical methods, you can calculate very limited types of European type options, the rest are calculated using numerical methods.

Numerical methods include the binomial method, the finite difference method and the Monte Carlo method.

Usually, the computations become more complicated as the dimension of the problem increases, i.e. a large number of variables, in such cases the Monte Carlo method is used.

The essence of this method is as follows: to estimate the mathematical expectation of payment, which is repeatedly simulated and calculated using a computer, the possible stock prices.

Usually, the solution to a stochastic differential equation is found using the Monte Carlo method too. A stochastic differential equation contains terms that are random variables. Solving such an equation is also a random process.

The application of the Monte Carlo method can be described by the following algorithm. Generate n random numbers, calculate the asset price for each value, find the arithmetic mean, and calculate the option price using the formula.

By increasing the number of random variables, the convergence to the exact value can be improved. In our paper we calculate the multi-dimensional integrals which is the solution of stochastic differential equations.

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NONLINEAR INTERVAL EIGENVALUE PROBLEMS IN FRACTURE MECHANICS

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In the problems of determining the stress and strain fields (strain rates) in the immediate vicinity of the crack tip in materials with nonlinear constitutive relations (power law of hardening, power law of the theory of steady-state creep), as well as when using power-law variants of evolutionary equations of damage accumulation in metals and structural alloys, the solution searched based on the variable separation method.

Determination of stress-strain fields requires solving a system of equations consisting of an equilibrium equation, a compatibility condition, and governing equations

$$r\sigma_{rz,r} + \sigma_{rz} + \sigma_{\theta z,\theta} = 0, \quad \varepsilon_{rz,\theta} - r\varepsilon_{\theta z,r} - \varepsilon_{\theta z} = 0, \quad \varepsilon_{i,j} = 3B\sigma_e^{n-1}s_{ij}/2, \tag{1}$$

where σ_{ij} are stress tensor components; ε_{ij} are the components of the strain tensor; $B,\ n$ - material constants determined experimentally; $s_{ij} = \sigma_{ij} - 1/3\sigma_{kk}\sigma_{ij}$ - components of the stress deviator; $\sigma_e = \sqrt{3/2s_{ij}}$ is the shear stress intensity; $r,\ \theta$ - polar coordinates with the pole at the apex of the defect.

The power-law character of the constitutive equations allows us to turn to the representation of the stress function near the crack tip in polar coordinates as an expansion in terms of eigenfunctions:

$$F(r,\theta) = r^{\lambda} f(\theta), \quad \sigma_{rz} = \frac{F_\theta}{r}, \quad \sigma_{rz} = -F_\theta, \quad \sigma_{\theta z} = -F_r, \tag{2}$$

where F is a function of stresses.

The deformation compatibility condition (1) leads to a second-order nonlinear ordinary differential equation with respect to the function $f(\theta)$

$$f''(\theta)\left[n(f'(\theta))^2 + \lambda^2 f^2(\theta)\right] + f\left[C_1(f'(\theta))^2 + C_2 f^2(\theta)\right] = 0, \tag{3}$$

where the notation

$$C_1 = \lambda(n-1)(2\lambda-1) + \lambda^2, \quad C_2 = \lambda^3(n-1)(\lambda-1) + \lambda^4. \tag{3}$$

The function $f(\theta)$ must satisfy the boundary condition, i.e. lack of surface conditions on the crack edges $f|_{\theta=\pm(\pi-\alpha)} = 0$.

In this paper, an interval method is considered that allows one to find an interval analytical solution to the problem. An interval-analytical expression for the eigenvalue λ, as an interval-valued function of the material nonlinearity index n and λ_0, the eigenvalue corresponding to the point linear problem (n = 1), is found using the formulas

$$\boldsymbol{\lambda} = [\lambda_0 - \varepsilon,\ \lambda_0 - \varepsilon], \quad n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \ldots,$$

$$f(\theta) = f_0(\theta) + \varepsilon f_1(\theta) + \varepsilon^2 f_2(\theta) + \ldots,$$

where ε is the deviation of the eigenvalue of the nonlinear problem from the eigenvalue λ_0, $f_0(\theta)$ refer to the linear "point" problem. An interval solution is obtained for a nonlinear eigenvalue problem following from the problem of determining the stress-strain state at the tip of an antiplane shear crack in a material with power-law constitutive relations.

ONE ONE-DIMENSIONAL INVERSE PROBLEM FOR THE HOPF-TYPE SYSTEM

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Let us consider the problem of the nonlinear waves propagation in a two-fluid medium taking into account the excitation source. In this case, the system of equations of two velocity hydrodynamics has the form [1-3]

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = -b(u_1 - u_2) + f(x)g(t), \tag{1}$$

$$\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = \varepsilon b(u_1 - u_2) + f(x)g(t), \tag{2}$$

where $f(x)g(t)$ is the source of signal excitation.

Let the Cauchy data

$$u_k|_{t=0} = u_k^0(x), \quad k = 1, 2, \tag{3}$$

and some additional information be given

$$u_1|_{x=0} = \varphi(t).$$

The inverse problem is to determine the function $(u_1(x,t), u_2(x,t), g(t))$ from equations (1)-(3). In this case, the function $f(x)$ is known and differs from zero, the matching condition is satisfied

$$u_1^0(0) = \varphi(0).$$

In this paper, we have proved the following theorem.

Theorem. Let $f(x), u_1^0(x), u_2^0(x) \in C^\omega(Y)$ be the class of real analytic functions, $f(x) \neq 0$. Then there are functions $u_1(x,t), u_2(x,t), g(t)$ that solve the inverse problem (1)-(3), satisfyed in some neighborhood of zero, such that in it $u_1(x,t), u_2(x,t), g(t) \in C^\omega$.

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NUMERICAL SIMULATION OF THE SEISMIC WAVES PROPAGATION FROM SINGULAR SOURCES IN POROUS MEDIA BASED ON SOLVING A LINEAR TWO-DIMENSIONAL PROBLEM

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Let us consider the formulation of the dynamic problem of the seismic waves propagation from singular sources in the media consisting of elastic and porous layers. In this case, the propagation of seismic waves in a porous medium saturated with a liquid in the absence of energy loss is described for the Cartesian coordinate system in a half-plane $x_2 \geq 0$ by the following initial-boundary value problem [1-3]:

$$\frac{\partial u_i}{\partial t} + \frac{1}{\rho_s}\frac{\partial \sigma_{ik}}{\partial x_k} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} = F_i(x_1,x_2)f(t), \quad \frac{\partial p}{\partial t} - (K - \alpha\rho\rho_s)\,\mathrm{div}\,\mathbf{u} + \alpha\rho\rho_l \mathrm{div}\,\mathbf{v} = 0,$$

$$\frac{\partial \sigma_{ik}}{\partial t} + \mu\left(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\right) + \left(\frac{\rho_l}{\rho}K - \frac{2}{3}\mu\right)\delta_{ik}\mathrm{div}\,\mathbf{u} - \frac{\rho_s}{\rho}K\delta_{ik}\mathrm{div}\,\mathbf{v} = 0, \quad \frac{\partial v_i}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x_i} = F_i(x_1,x_2)f(t),$$

$$u_i|_{t=0} = v_i|_{t=0} = \sigma_{ik}|_{t=0} = p|_{t=0} = 0, \sigma_{22} + p|_{x_2=0} = \sigma_{12}\,|_{x_2=0} = \left.\frac{\rho_{0,l}}{\rho_0}p\right|_{x_2=0} = 0,$$

where $\mathbf{u} = (u_1,u_2)$ and $\mathrm{div}\,\mathbf{v} = (v_1,v_2)$ are the velocity vectors of an elastic porous body with the partial density ρ_s and the liquid with a partial density ρ_l, respectively, p is the pore pressure, σ_{ik} is the stress tensor $\rho = \rho_l + \rho_s$, $\rho_s = \rho_s^f(1-d_0)$, $\rho_l = \rho_l^f d_0$, ρ_s^f, and ρ_l^f are the physical densities of the elastic porous body and the liquid, respectively, d_0 is the porosity, δ_{ik} is the Kronecker symbol, $K = \lambda + 2\mu/3, \lambda > 0$, $\mu > 0$ are the Lame coefficients , $\alpha = \rho\alpha_3 + K/\rho^2$, $\rho^3\alpha_3 > 0$ is the modulus of volumetric compression of the liquid component of the heterophase medium, $\mathrm{div}\,\mathbf{F} = (F_1, F_2)$ is the vector of mass forces, $f(t)$ is the simulated time signal in the source. Here F_1 and F_2 are the components of the force vector describing the action of a source localized in space.

The medium is considered to be ideal (there is no energy loss in the system) isotropic and two-dimensional inhomogeneous with respect to space. For the numerical solution of the problem posed, the method of integrating the integral Laguerre transform with respect to time with finite difference approximation in spatial coordinates is used. The solution algorithm employed makes it possible to efficiently carry out simulations in a complex porous medium and to study the wave effects arising in such media.

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A NEW MIXED VARIATION PROBLEM AND STOKES SYSTEM WITH THE DELTA SOURCE

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Let Ω be a two-dimensional bounded simply connected domain with a sufficiently smooth boundary $\partial\Omega$. The stationary Stokes problem of the motion of a viscous incompressible fluid is considered

$$-\nu\mathbf{\Delta u}+\nabla p=\mathbf{f},\quad \mathrm{div}\mathbf{u}=0\quad \text{in } \Omega,$$

$$\mathbf{u}=\mathbf{0}\qquad \text{on } \partial\Omega, \tag{1}$$

where $\mathbf{f}\in\mathbf{H}^{-1-\varepsilon}(\Omega), 0<\varepsilon<1/2$, $\nu>0$, $\mathbf{H}^s(\Omega)=(H^s(\Omega))^2$, $\mathrm{H}^s(\Omega)$ denotes the Sobolev Hilbert space of order s with the norm $\|\cdot\|_{s,\Omega}$ and the seminorm $|\cdot|_{s,\Omega}$ [1].

Let X_i, M_i, i = 1,2 be four real Hilbert spaces equipped with the scalar products $(\cdot,\cdot)_{X_i}$, $(\cdot,\cdot)_{M_i}$. We introduce three continuous bilinear forms [5]: $a(\cdot,\cdot): X_1\times X_2\to\mathbb{R}$, $b_1(\cdot,\cdot): X_2\times M_2\to\mathbb{R}$, $b_2(\cdot,\cdot): X_1\times M_1\to\mathbb{R}$, for which the inequalities hold

$$a(u,v)\le C\|u\|_{X_1}\|v\|_{X_2}\quad b_1(v,p)\le C\|v\|_{X_2}\|p\|_{M_2},\quad b_2(u,q)\le C\|u\|_{X_1}\|q\|_{M_1}$$

$$\forall u\in X_1,\,\forall v\in X_2,\quad \forall v\in X_2,\,\forall p\in M_2,\quad \forall u\in X_1,\,\forall q\in M_1.$$

Variational problem. Given $F\in X_2'$ and $G\in M_1'$, it is required to find a pair $(u,p)\in X_1\times M_2$ such that

$$a(u,v)+b_1(v,p)=<F,v>\quad b_2(u,q)=<G,q>\quad \forall v\in X_2,\quad \forall q\in M_1. \tag{2}$$

The following statements are proved in the paper

Lemma 1. Let a triple of the Hilbert spaces V,H,V' be a framing of the space H. Then for every element $v\in V$ there exists a functional $f\in V'$ such that

$$\|v\|_V=\|f\|_{V'}\quad \text{and}\quad (v,f)_H=\|v\|_V^2.$$

Lemma 2. Let the bounded open domain $\Omega\subset\mathbb{R}^2$ satisfy the cone condition. Then (i) the operator ∇^ε is an isomorphism from $H_\perp^\varepsilon(\Omega)$ to V_ε^0; (ii) the operator div^ε is an isomorphism from $V_\varepsilon^\perp$ to M_2.

Let be $X_1=\mathbf{H}_0^{1-\varepsilon}(\Omega), X_2=\mathbf{H}_0^{1+\varepsilon}(\Omega)$. It is required to find a pair $(\mathbf{u},p)\in X_1\times M_2$:

$$a(\mathbf{u},\mathbf{v})+b_1(\mathbf{v},p)=<\mathbf{f},\mathbf{v}>_{-1-\varepsilon,1+\varepsilon}\quad b_2(\mathbf{u},q)=0\quad \forall\mathbf{v}\in X_2,\quad \forall q\in M_1, \tag{3}$$

Theorem 2. Stokes problem (1) when $\mathbf{f}\in\mathbf{H}^{-1-\varepsilon}(\Omega)\,(0<\varepsilon<1/2)$, has a unique generalized solution $(\mathbf{u},p)\in\mathbf{H}_0^{1-\varepsilon}(\Omega)\times\left(H_\perp^\varepsilon(\Omega)\right)'$ as a solution to the mixed variational problem (3), and it satisfies the estimate

$$\|\mathbf{u}\|_{1-\varepsilon,\Omega}+\|p\|_{-\varepsilon,\Omega}\le C\|\mathbf{f}\|_{-1-\varepsilon,\Omega},$$

where $C>0$ is a positive constant independent of $\mathbf{f}$.

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THE PROBLEM OF DETERMINING TIME-DEPENDENT DARCY COEFFICIENT FROM THE POROELASTICITY SYSTEM

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Let D there be an interval $(0,1)$, there Q is a rectangle. Further, let $f(x,t)$, $K(x,t)$, $u_0(x)$, $u_1(x)$ and $\mu(t)$ there are functions given at $x \in [0,1]$, $t \in [0,T]$.

Inverse problem. Find functions $u(x,t)$, $v(x,t)$ and, $s(t)$ connected in a rectangle by a system of equations [1-5]

$$u_{tt} - u_{xx} + \gamma\, s(t)\,(u_t - v_t) = f(x,t), \quad v_{tt} - s(t)\,(u_t - v_t) = f(x,t), \tag{1}$$

when the boundary conditions are fulfilled for the function $u(x,t)$

$$u(0,t) = u(1,t) = 0, \quad 0 < t < T \tag{2}$$

and fulfillment for functions $u(x,t)$ and $v(x,t)$ initial conditions

$$u(x,0) = u_0(t), \quad u_t(x,0) = u_1(t), \quad x \in D \tag{3}$$

$$v(x,0) = 0, \quad v_t(x,0) = 0, \quad x \in D \tag{4}$$

with the additional condition

$$\int_0^1 K(x,t)\,u(x,t)dx = \mu(t), \quad t \in (0,T). \tag{5}$$

In system (1) $u(x,t)$ and $v(x,t)$ are the components of the displacement velocity of an elastic porous body and a saturating liquid by the corresponding constant partial densities ρ_s and, ρ_s. In this paper, for simplicity, we assume that the propagation of a shear wave is constant and equal to unity, $\gamma = \rho_l/\rho_s$, the function of the form characterizes the Darcy coefficient and is responsible for energy dissipation in system.

In this paper, the solvability of the inverse problem (1) - (5) in the Sobolev space has been shown.

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PROPAGATION OF NON-STATIONARY SHEAR WAVES FROM A THICK-WALLED SHELL IN A POROUS-ELASTIC SPACE

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The article considers the problem of the propagation of unsteady transverse waves from a thick-walled elastic spherical shell in a porous-elastic space. Mathematical modeling and study of non-stationary wave processes in continuous media is a complex and important direction of wave dynamics in the mechanics of a deformable solid. Problems of the propagation and diffraction of non-stationary shear waves in elastic bodies are of great theoretical and practical importance in such fields of science and technology as seismic exploration of minerals, seismic resistance of structures, and many others.

The introductory part of the article provides a detailed analysis of the literature on the subject.

Let a thick-walled elastic spherical shell with inner and outer radius R_1 and R_2 ($R_1 < R_2$), respectively, be located in a linearly porous-elastic homogeneous isotropic space

The motion of media is considered in a spherical coordinate system (r, θ, ϑ) with the origin at the center of the shell. At the initial moment of time $\tau = 0$, the shell is in an unperturbed state. On the inner surface of a thick-walled spherical shell, two types of boundary conditions are considered.

Task 1. An axisymmetric specified tangential surface load $q(\tau, \theta)$ is applied to the inner surface of the shell.

Task 2. On the inner surface of the shell, a tangential displacement $V(\tau, \theta)$.

The contact conditions of the media, consisting in the continuity of displacement and stress.

Taking into account the axial symmetry, the problems of motion of the shell and the environment relative to the elastic potentials ψ_l are described by the wave equations and the initial conditions are homogeneous. There is no disturbance at infinity.

To solve the problem, the integral Laplace transform in dimensionless time and the method of incomplete separation of variables were used. In the image space, the problem is reduced to an infinite system of linear algebraic equations, the solution of which is sought in the form of an infinite exponential series. Formulas for the components of the displacement vector and the stress tensor are obtained. The transition to the originals is carried out using the theory of residues. Numerical experiments have been carried out, the results of which are presented in the form of graphs. The obtained results of the work can be used in the field of geophysics, seismology and design organizations in the construction of structures, as well as in the design of underground reservoirs.

EFFECT OF THE EXTERNAL FIELD ON SOME ENTANGLEMENT MEASURES OF QUBIT INTERACTION WITH A BIMODAL ENTANGLED FIELD

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Abstract. The influence of external classical field on interaction between a correlated two modes of electromagnetic field and a three-level atom in Λ structure is studied. Using the Schrödinger equation, the general solution to the wave equation is obtained. The effect of the external classical field on the phenomena of collapses and revivals, squeezing and atomic marginal distribution are discussed. In our analysis, we assume that the field prepared in the pair coherent state and the atomic system in the

upper-most state. The results confirmed that the atomic level occupation is affected by the addition of the classical field. The presence of the classical field reduce the periods of the entropy and variance squeezing. The classical field reduces the extreme values of the atomic marginal distribution.

PROPERTIES OF MATHEMATICAL MODELS DESCRIBED BY THE DOUBLE NONLINEAR HEAT TRANSFER EQUATION WITH VARIABLE DENSITY AND A ABSORPTION

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In this paper, we study the properties of mathematical models described by the double nonlinear heat transfer equation with variable density and a absorption.

Consider in the domain $Q = \{(t, x) : t \in R_+,\ x \in R^2\}$ problem Cauchy for double nonlinear parabolic system

$$\begin{cases} |x|^k \frac{\partial u}{\partial t} = div\left(|x|^n u^{m_1-1} |\nabla u|^{p-2} \nabla u\right) + \gamma(t) |x|^k u^{q_1} v^{r_1} \\ |x|^k \frac{\partial v}{\partial t} = div\left(|x|^n v^{m_2-1} |\nabla v|^{p-2} \nabla v\right) + \gamma(t) |x|^k u^{q_2} v^{r_2} \end{cases} \tag{1}$$

$$u(0, x) = u_0(x) \geq 0,\ v(0, x) = v_0(x) \geq 0, \tag{2}$$

where $m_1 > 1$, $m_2 > 1$, $p > 2$, q_1, q_2, r_1, $r_2 \geq 1, n \geq 1$, $k \leq 0$ is the parameters, $u_0(x)$, $v_0(x)$ is the initial conditions, $|x|^n$, $|x|^k$ is the density of the medium, $0 < \gamma(t) \in C(0, \infty)$ is the specified function.

The motivation for considering this problem it is a degenerate partial differential equation and therefore is a source for the emergence of new nonlinear effects such as finite velocity propagation of perturbation, a spatial localization of bounded and unbounded solutions to the emergence of which were first established in the works [1] for the particular value of the numerical parameters when $q_1 = 0, r_1 = 0$.

System (1) in the case p=2 is called porous media equation different properties of solution studied huge number of works (see for example [1-4] and references therein).

The system (1) in the case $m_i + p - 3 > 0,\ i = 1, 2$ is called slowly diffusion and when $m_i + p - 3 < 0,\ i = 1, 2$ it called the fast diffusion equation. An asymptotic solution is usually understood as a solution of a system of nonlinear equations that can satisfy certain conditions. The case $m_i + p - 3 = 0$, $i = 1, 2$ we call a double critical case.

In this talk the qualitative properties of solution of the problem (1) depending on value of numerical parameters and initial data are considered. Estimates of solution asymptotic of self-similar solutions, the numerical aspects considered problem are discussed.

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NUMERICAL MODELING OF A NONLINEAR HEAT CONDUCTION PROBLEM BY THE FOURIER METHOD

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In the paper it is considered systems of nonlinear nonstationary equations of two-dimensional heat conduction of the form [1]:

$$\frac{\partial u_i}{\partial t} = u_{3-i}^{n_i} \Delta u_i - f_i(x_1, x_2, u_1, u_2, t); \tag{1}$$

in the area of $Q = \{a \leq x \leq b, c \leq y \leq d, 0 \leq t \leq T\}$ for cases of boundary conditions of the first kind:

$$u_i = (a, y, t) = \varphi_{i1}; u_i = (b, y, t) = \varphi_{i2}; u_i = (x, c, t) = \varphi_{i3}; u_i = (x, d, t) = \varphi_{i4}; \tag{2}$$

which are further generalized for the remaining cases of boundary conditions, and the initial conditions

$$u_i|_{t=0} = \Psi_i(x_1, x_2); \tag{3}$$

where: n_i-known physical constants; in particular: $x_1 = x; x_2 = y;$
To solve this problem (1) $-$ (3) the iterative method of alternating directions is applied, a combination of the sweep method[2]. As an initial approximation, we take stationary solutions of the equations of the equations $\Delta u_i = 0$, under boundary conditions (2) by the Fourier method, of the form:

$$u_i = \frac{\varphi_{i2} \cdot \sin \lambda(x-a) + \varphi_{i1} \cdot \sin \lambda(b-x)}{\sin \lambda(b-a)} \cdot \frac{(\varphi_{i4} \cdot e^{\lambda(d-c)} - \varphi_{i3})e^{\lambda(y-c)} + (\varphi_{i3} \cdot e^{\lambda(d-c)} - \varphi_{i4})e^{\lambda(d-y)}}{(e^{2\lambda(d-c)} - 1)}$$

where $\lambda(b-a) \neq k\pi$ $(k = 0, 1, 2, ...)$. An analysis of the obtained results of a computational experiment for a test problem with various methods of linearization and choice of an initial approximation is given.

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ON THE STEADY MOTIONS GLOBAL STABILIZATION OF A FIVE-LINK MANIPULATOR

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In the early 90s of the last century, the problem of controlling the movement of manipulative robots without measuring speeds began to be actively studied. This study was associated with a number of facts: the operation of the coordinate sensors is associated with the occurrence of noise that worsens the control accuracy, the installation of additional sensors in the manipulators leads to a significant increase in their cost and other problems [1, 2].

The purpose of this work is to substantiate the control models that provide the global stabilization of a five-link manipulator without measuring velocities.

The position of the manipulator under consideration is determined by five coordinates: $\theta_1, \theta_2, \theta_4$, θ_5 are the angles of rotation of the links around the cylindrical joints and the gripper, d_3 is the longitudinal movement of the link with the prismatic hinge. In this case, the base link is directed vertically.

The Lagrange motion equations are given by [1]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + d(q,\dot{q}) = u \tag{1}$$

where $q = (\theta_1, \theta_2, d_3, \theta_4, \theta_5)'$ is the vector of joint positions, $M(q) \in R^{5\times5}$ is a positive-definite inertia matrix, $C(q,\dot{q}) \in R^{5\times5}$ is the matrix of Coriolis and centrifugal forces, $g(q) \in R^5$ is the vector of gravitational forces and torques, $d(q,\dot{q}) \in R^5$ is the vector of viscous damping forces and torques, $u \in R^5$ is the control vector.

If $u = u^{(0)} = g(q^{(0)})$, then the manipulator (1) has the equilibrium position $\dot{q} = 0$, $q = q^{(0)}$. Let $x = q - q^{(0)}$.

On the base of [3], we prove that the controller without velocity measurement

$$u^{(1)} = u - u^{(0)} = -k\arctan x + \frac{1}{1+x^2}\int\limits_{t-h}^{t}\frac{\arctan x(\tau)}{h+1+\tau+t}d\tau,\ \ k,h>0$$

solves the problem on global stabilization of this position.

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MODELING VIBRATION OF COMPOSITE PIPELINES CONVEYING FLUID

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Pipeline transport is the main and most frequently used means to convey various fluids over long distances. It is the only type of transport that conveys the transported product, while remaining in a stationary position. Despite the advantages and the widespread prevalence of this type of transport, trunk pipelines have all the signs of a source of increased danger, and the unexpected vibration of the pipeline caused by various external and internal factors limits their use. Therefore, vibration of pipelines is of scientific and practical interest and attracts the attention of scientists and specialists The effect of non-stationary external forces on the vibration of pipelines made of composite materials is investigated in the paper. A mathematical model of composite pipelines vibration is developed taking into account the viscosity properties of the structure and pipeline bases material, axial forces, internal pressure, resistance forces and external disturbances. A mathematical model of viscoelastic pipelines conveying fluid under vibrations is constructed based on the Boltzmann–Volterra integral model. The mathematical model to study a pipeline is based on the Euler-Bernoulli theory of beam. Considering physicomechanical properties of the pipeline material, the mathematical model of the problems under consideration presents a system of integro-differential equations in partial derivatives with corresponding initial and boundary conditions. The nonlinear partial differential equations, obtained using the Bubnov–Galerkin method under considered boundary conditions, are reduced to solving the system of ordinary integro-differential equations. A computational algorithm is developed based on the elimination of features of integro-differential equations with weakly singular kernels, followed by the use of quadrature formulas. It is shown that the error of the method coincides with the error of the used quadrature formulas and has the same order of smallness with respect to the interpolation step. The effect of a non-stationary external load on the behavior of a viscoelastic pipeline is analyzed. It is found that with an increase in external load parameter, vibration amplitudes increase, and vibration frequency decreases. It is revealed that with an increase in the frequency of non-stationary external loads, the amplitude and frequency of pipeline vibration increase.

MODELING OF NONLINEAR PROCESSES BY SOLVING AN EQUATION OF HAMILTON-JACOBI TYPE

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In this talk study $Q_T = \{(t, x) : 0 < t < T, x \in R_+\}$ the qualitative properties of solutions based on a self-similar analysis to the following problem for the heat equation with double nonlinearity [1].

$$L(u) = -\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(u^{m-1}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\frac{\partial u^k}{\partial x}\right) = 0 \tag{1}$$

$$u|_{t=0} = u_0(x) \geq 0, u|_{x=0} = (T-t)^{-\alpha}, 0 < t < T, \alpha > 0 \tag{2}$$

where $k \geq 1, m, p \geq 2$ are numerical parameters characterizing the property of a nonlinear medium. The first boundary condition is called the blow-up regime. In this work qualitative properties of the problem established using the solution of corresponding to the equation (1) the Hamilton- Jacobi equation.

$$\frac{\partial u}{\partial t} = ku^{m+k-3}\left|\frac{\partial u^k}{\partial x}\right|^{p-2}\left(\frac{\partial u}{\partial x}\right)^2, u(0,x) = u_0(x) \geq 0, x \in R_+ \tag{3}$$

We consider the following self-similar solution of the equation (1)

$$u(t,x) = (T-t)^{-\alpha} f(\xi), \ \xi = x\,[\tau(t)]^{-\frac{1}{p}}, \tau(t) = (T-t)^{1-(m+k(p-1)-2)\alpha}, \alpha > 0 \tag{4}$$

where function $f(\xi)$ satisfy to the self-similar equation

$$f^{m+k(p-1)-2}\left|\frac{df^k}{d\xi}\right|^{p-2}\left(\frac{df}{d\xi}\right)^2 - \left(\frac{1-(m+k(p-1)-2)\,\alpha}{p}\right)\xi\frac{df}{d\xi} + \alpha f = 0 \tag{5}$$

which have the following approximately solution

$$f(\xi) = (b - \xi^{\gamma})_+^{\gamma_1},$$

where $(n)_+ = max(0, n), b = const. \geq 0, \gamma_1 = \dfrac{p-1}{m+k(p-1)-2}, \gamma = \dfrac{p}{p-1}$

Theorem 1. Let, $1 < m + k(p-1) - 1 \leq \dfrac{1}{\alpha}$. Then the solution of problem (1)-(2) is spatially localized.

At the numerical solution of a problem iterative processes were constructed basing on the method Picard, Newton. Results of computational experiments shows, that all listed iterative methods are effective for the solution of considered nonlinear problems and leads to the nonlinear effects if we will use as initial approximation the solutions of Hamilton-Jacobi [2].

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NONLINEAR DYNAMICS OF QUANTUM ENTANGLEMENT

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Abstract. In this work, we will examine in a proof-of-concept experiment a new type of quantum-inspired protocol based on the idea of nonlinear dynamics of quantum entanglement. We discuss various measures of bipartite and tripartite entanglement in the context of two and three level atoms. The quantum entanglement is discussed for different systems. For the three-level systems various measures of tripartite entanglement are explored. The significant result is that a sudden death and sudden birth of entanglement can be controlled through the system parameters.

DEVELOPMENT OF A REGRESSION MODEL BASED ON STATISTICAL ANALYSIS OF THE VOLUME OF FOREIGN DIRECT INVESTMENT IN DEVELOPING COUNTRIES

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Foreign direct investment is important for the development of the economy of every country. This is especially true for the economies of developing countries. In this analysis, a regression model is built on the factors that influence FDI flows to 30 developing countries. The statistics used in this analysis were based on data from the 2020 World Bank database. https://data.worldbank.org/
The following statistics were used for the study: Foreign direct investment Y (in USD) was taken as the dependent variable. The main factors that directly affect the volume of foreign investment in the country's economy are:
-GDP growth rate was taken as X_1. As GDP increases, so does investment (unit: interest (%));
-The level of trade openness (relative to export and import GDP) was taken as X_2. The level of foreign investment and trade openness has a positive correlation (unit percentage: (%));
-Gross domestic reserves (as a percentage of GDP) were taken as X_3. Increases in fixed capital include changes in turnover and cash reserves, as well as expenditures on the acquisition of assets (combined: interest (%));
-Real interest rates were taken as X_4. This indicates what part of the investment the interest payment is equal to, ie the share in it (unit: interest (%));
-The inflation rate was taken as X_5. Inflation is the devaluation of money (unit:%) as a result of a sharp rise in prices;
-The unemployment rate was taken as X_6. This is the level of the non-employed workforce (unit: percentage (%));
-The labor force was taken as X_7. It is the physical and mental ability of a person to work (unit: number);
-X_8 is the share of exports of goods and services in GDP (unit: percent (%));
-X_9 as gold - foreign exchange reserves - financial assets held by the country's governments and central banks (unit: US dollars);
Based on the above statistics, a correlation and regression analysis was performed and a regression model for foreign direct investment was developed:

$$Y = -9,6 \cdot 10^5 \cdot X_2 + 3,44 \cdot 10^6 \cdot X_4 + 191,1 \cdot x_7 + 0,0615 \cdot X_9$$

and for this model, T test, F test, and 95% confidence intervals were constructed.

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CAUCHY PROBLEM FOR THE QUASILINEAR EQUATION IN NON-DIVERGENT FORM WITH SOURCE IN NON-HOMOGENEOUS MEDIUM

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We consider the Cauchy problem for the quasilinear equation in non-divergent form with density

$$u_t = u^q \nabla \left(|x|^n u^\sigma \nabla u\right) + t^m |x|^{-l} u^p \tag{1}$$

$$u\left(t,x\right)|_{t=0} = u_0(x) \geq 0, x \in \mathbb{R}^N \tag{2}$$

$q \geq 0, n, \sigma, m, l, p \neq 1$ are numerical parameters. Problem (1)-(2) represents a series of physical processes: the process of heat dissipation, the filtration process of liquids and gases in non-homogeneous medium and other nonlinear processes.
We study the self - similar solutions [1] of the equation (1) in the form:

$$u\left(t,x\right) = t^{-\alpha} \overline{f}\left(\xi\right)$$

where $\xi = |x|t^{-\beta}, \alpha = \dfrac{(m+1)(n-2)+l}{(p-1)(n-2)+l(q+\sigma)}, \beta = \dfrac{(m+1)(q+\sigma)+1-p}{(p-1)(n-2)+l(q+\sigma)}$,

$$\overline{f}\left(\xi\right) = A\xi^\gamma \tag{3}$$

$\gamma = \dfrac{l+n-2}{p-q-\sigma-1}, \quad A = \left[\gamma\left(2-n-\gamma\left(\sigma+1\right)-N\right)\right]^{\dfrac{1}{p-q-\sigma-1}}$

If $\gamma\left(q+\sigma\right) > 2-n$ inequality in force then, (3) will be an asymptotic approximation solution.
Let, $\varepsilon = sgn\left(A\left(\alpha+\beta\gamma\right)\right)$. Then $\overline{f}$ becomes:

- a sup(sub) solution, at ε = -1 ($\varepsilon = 1$) ;
- an analytical solution at $\varepsilon = 0$.

The results of the computational experiments shows that the self-similar solutions more appropriate and the iterative method based on the Newton method is efficient for the solution of nonlinear problems and keep the nonlinear effects if we use as initial approximation the solutions of the self-similar equations constructed by the method of nonlinear splitting [1].

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ASYMPTOTICS OF THE SOLUTION A SYSTEM OF THE SEMILINEAR HEAT CONDUCTION PROBLEM WITH ABSORPTION AT A CRITICAL EXPONENT

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Consider in the domain $Q = \{(t,x) : t > 0, x \in R^N\}$ the following system of semilinear heat conduction equations

$$L_1(u) \equiv -|x|^{-l}\frac{\partial u}{\partial t} + \Delta(|x|^n u) - |x|^{-l} v^{\beta_1} = 0,\ L_2(v) \equiv -|x|^{-l}\frac{\partial v}{\partial t} + \Delta(|x|^n v) - |x|^{-l} u^{\beta_2} = 0 \qquad (1)$$

$$u(0,x) = u_0(x) \geq 0, v(0,x) = v_0(x) \geq 0, x \in R^N \qquad (2)$$

where t and x are, respectively, the temporal and spatial coordinates, $\beta_1, \beta_2 > 1,\ l > 0, n > 0,\ \Delta = \sum_{i=1}^{N} \partial^2/\partial x_i^2$ Problem (1)-(2) is the basis for modeling various processes of nonlinear heat diffusion, magnetic hydrodynamics, gas and liquid filtration, oil and gas, in the theory of non-Newtonian fluids, etc. M. Herrero, M. Escobedo [1] establish the following condition Fujita type global solvability

$$(\beta_i + 1)/(\beta_1\beta_2 - 1) > N/2,\ i = 1, 2$$

to the problem (1) when l=0, n=0 In this work the following value of critic exponent of Fujita type

$$\frac{\beta_i + 1}{\beta_1\beta_2 - 1} = s/2,\ i = 1, 2$$

where $s = \frac{2(N-l)}{2-l-n},\ l < N,\ l + n < 2$ which consist the result of the work [1] established. It is proved that asymptotic solution of the problem (1),(2) for large time have

$$u(t,x) \sim H((T+t)\ln(T+t))^{-s/2} e^{-\frac{\varphi(x)^2}{4(T+t)}}, v(t,x) \sim H_1((T+t)\ln(T+t))^{-s/2} e^{-\frac{\varphi(x)^2}{4(T+t)}}$$

$\varphi(x) = 2(2-l-n)^{-1}|x|^{\frac{2-l-n}{2}}$,where H and H1 some positive constants. The obtained asymptotic of solutions are used as an initial approximation in computational experiments. Results of numerical experiments showed the effectivity of suggested numerical scheme and the method of solution. Analogy of this result for one equation case was established in [2].

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A HYBRID FRACTIONAL OPTIMAL CONTROL FOR A NOVEL CORONAVIRUS (2019-NCOV) MATHEMATICAL MODEL

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Abstract. COVID-19 pandemic is the defining global health crisis of our time and the greatest challenge we have faced since world war two. To describe this disease mathematically, we noted that COVID-19, due to uncertainties associated to the pandemic, ordinal derivatives and their associated integral operators show deficient. The fractional order differential equations models seem more consistent with this disease than the integer order models. This is due to the fact that fractional derivatives and integrals enable the description of the memoryand hereditary properties inherent in various materials andprocesses. Hence there is a growing need to study and use the fractional order differential equations. Also, optimal control theory is very important topic to control the variables in mathematical models of infectious disease.

Moreover, a hybrid fractional operator which may be expressed as a linear combination of the Caputo fractional derivative and the Riemann–Liouville fractional integral is recently introduced. This new

operator is more general than the operator of Caputo's fractional derivative. Numerical techniques are very important tool in this area of research because most fractional order problems do not have exact analytic solutions.

In this talk, a novel fractional order Coronavirus (2019-nCov) mathematical modelwith modified parameters is presented. The new fractional operator can be written as a linear combination of a Riemann–Liouville integral and a Caputo derivative. The suggested system is ruled by eight fractional-order nonlinear differential equations. We show that our COVID-19 model describes well the real data of daily confirmed cases in Wuhan.The optimal control of the suggested model is the main objective of this work. Three control variables are presented in this model to minimize the number of infected population. Necessary control conditions are derived. Two schemes are constructed to simulate the proposed optimal control system. Prove of the schemes- stability are given. In order to validate the theoretical results numerical simulations and comparative studies with Caputo derivative are given.

NONLINEAR SPLITTING ALGORITHM FOR A PARABOLIC SYSTEM OF QUASI-LINEAR EQUATIONS

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Let's consider $Q = \{(t,x) : 0<t<\infty, x \in R^N\}$ parabolic system of two quasi-linear reaction-diffusion equations

$$\begin{cases} \frac{\partial u_1}{\partial t} = \nabla\left(|x|^n u_1^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1\right) + a_1(t)u_1 - b_1(t)u_2^{\beta_1}, \\ \frac{\partial u_2}{\partial t} = \nabla\left(|x|^n u_2^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2\right) + a_2(t)u_2 - b_2(t)u_1^{\beta_2}, \end{cases} \tag{1}$$

$$u_1\left|_{t=0} = u_{10}(x)\right., u_2\left|_{t=0} = u_{20}(x)\right. \tag{2}$$

which describes the process of a biological population of the Kolmogorov-Fisher type in a nonlinear two-component medium, whose diffusion coefficients are respectively $|x|^n u_1^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1$,$|x|^n u_2^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2$. Members $a_i(t)u_i$ $i = 1,2$ means population growth, and $b_i(t)u_i^{\beta_1}$ *power of extinction. Numerical parameters* $m_1, m_2, n, p, \beta_1, \beta_2$- are positive real numbers, $\nabla(.) - \underset{x}{grad}(.)$,$\beta_1, \beta_2 \geq 1$, $x \in R^N$; $u_1 = u_1(t,x) \geq 0$, $u_2 = u_2(t,x) \geq 0$ population density.

The biological population problem both for the case of a single equation and a system has been intensively studied by different authors (see [1-4] and literature cited there). In this paper we obtained an estimation of solutions of problems (1), (2) and the front of advance of individuals for the cases $k(p-2)+m_i-1>0$, $k(p-2)+m_i-1<0$, $k(p-2)+m_i-1=0$, $i = 1,2, \beta_1 \geq 1, \beta_2 \geq 1$ k(p-2)and n<p and the singular case n=p on the basis of the analysis of an approximate self-similar system of equations constructed by the method of nonlinear splitting and reference equations (1). The problem of the choice of initial approximation for the iteration process in the numerical solution of the considered problem solved. The results of numerical experiments showed the efficiency of the proposed approach.

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ANALYSIS OF PHYSICALLY NONLINEAR STRAIN IN TRANSVERSALLY ISOTROPIC BODIES

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The article provides mathematical modeling and research of physically nonlinear strain in transversely isotropic bodies of complex configuration. The influence of the configuration of stress concentrators on the elastoplastic state of structural elements made of fibrous composite materials is analyzed in a three-dimensional formulation.

To solve the problem of the physically nonlinear strain of fibrous composites, a simplified transverse-isotropic theory was used. It makes it possible to apply the theory of small elastoplastic strains to solve specific applied problems [1]:

$$\tilde{\sigma} = (\lambda_2 + \lambda_4)\,\tilde{\theta} + \lambda_3 \varepsilon_{33}; \quad \sigma_{33} = \lambda_3 \tilde{\theta} + \lambda_1 \varepsilon_{33}; \quad P = P(p); \quad Q = Q(q).$$

Here $P = 2\lambda_4\,(1 - \pi(p))\,p$, $Q = 2\lambda_5\,(1 - \chi(q))\,q$, $\pi(p)$ and $\chi(q)$ – are the plasticity functions of the Ilyushin type, their values in the elastic zone are zero. In the elastic region, the parameters of σ_{ij} are determined by Hooke's law, and in the region of plastic strains they are determined on the basis of the A. A. Ilyushin theory of plastic strains. The zones of plastic deformations are determined by the von Mises criterion.

To describe the anisotropy of the mechanical properties of fibrous materials, a strtural-phenomenological model is used, which makes it possible to represent the initial material in the form of a complex of two jointly working isotropic materials: the base material, considered from the standpoint of continuum mechanics, and the material of fibers oriented along the direction of anisotropy of the base material. At that, it is assumed that parallel fibers perceive only axial tensile-compression forces and are deformed together with the base material.

To determine the effective mechanical parameters of fibrous materials, relations are used that allow considering the internal structure of the material for calculating periodically nonhomogeneous materials based on the asymptotic averaging method and suitable for any values of properties and volume fractions of components [2].

On the basis of the developed algorithm for solving the problem, software was implemented, which allows performing various calculations for the numerical analysis of the stress-strain state of elements of spatial structures. By means of a computational experiment, it was determined that at volume content of fibers in the composite from 35% to 60%, the physical properties of boron aluminum are manifested; this ensures the joint operation of the fiber and matrix. The presence of an elastoplastic matrix provides a high strength of fibers loading. For various configurations of holes in boron aluminum, the location of the regions with the maximum values of shear stresses, where the separation of high-strength fibers from the matrix is possible, was defined.

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WAVE FORMATION IN CURRENTS ABOVE A SANDY BOTTOM

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Nowadays, much attention is paid to the study of the hydrodynamics of natural processes, where the main role is played by the use of mathematical modeling methods. Issues of design and stable safe operation of hydraulic structures are important both in theoretical and applied aspects. In this regard, numerical modeling of currents in rivers and canals, as well as in furrows with a washed-out sandy bottom, is of particular theoretical and practical interest. With such currents, the Froude number plays the main role. At sufficiently high speeds, the sandy bottom is washed out and the flow becomes chaotic (turbulent). With a calm current, the reformation of the sandy bottom is also observed. The wave structures formed in such flows are divided into two groups according to the mechanism of formation and their characteristics. The first includes wave motions associated with the presence of a free flow surface. They also exist when the fluid flows over a non-deformable bottom.These waves are formed at sufficiently large numbers Fr.

The applied mathematical models differ in the way they describe the movement of liquid and bottom sediments. The formation of waves on the lower surface of the flow was obtained using the model of the potential flow of an ideal fluid within the framework of systems of hydraulic equations or using semi-empirical models of turbulence [**?**]. In the potential and hydraulic models, the instability of the bottom surface was determined by the introduction of an artificial shift between the flow rate of bottom particles and the local characteristics of the fluid flow. In this paper, we propose a mathematical model for the flow of an incompressible viscous fluid with a free surface and a sandy bottom, which includes both mechanisms of instability and does not contain artificial phase displacement[**?**].

In this research we propose a mathematical model for the flows of a viscous incompressible fluid along an inclined plane under the action of gravity over a sandy bottom. The analysis was carried out in the linear instability of the flow and reduced the problem to finding the eigenvalues. Investigated and analyzed the questions of wave formation on the free and bottom surfaces of the liquid. The regions of flow instability are determined for any Froude numbers.

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MONTE CARLO SOLUTION OF DIRICHLET PROBLEM FOR SEMILINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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With the development of computer technologies, there is an increasing interest in numerical methods of solving applied problems, in particular, in statistical modeling (the Monte Carlo method). Historically, the intensive development of the theory and applications of the Monte Carlo method was associated with the solution of the actual problems of the theory of radioactive transfer.

Numerical statistical modeling is usually considered as the realization with the help of a computer of the probabilistic model of some object with the purpose of evaluating the studying characteristics based on the law of large numbers. The Monte Carlo method is normally applied to the modeling of any process, the flow of which is influenced by random factors, for example, in solving problems of statistical physics, the theory of turbulence, and the theory of transfer. When solving many mathematical problems that are not connected with random factors, we can use the method of statistical modeling by creating artificial probabilistic model of the problem solution(see [1]).

In this paper we introduce Monte Carlo methods to compute the solution of elliptic equations with pure Dirichlet boundary conditions. We consider the Dirichlet problem for the quasilinear Helmholtz equation. We obtained the probabilistic representations of the solutions of problems in the form of a mathematical expectation of some random variables and branching processes are constructed in accordance with probabilistic representations, simulating formulas for branching processes are given too. It is proved that the constructed branching processes degenerate with probability one and the average number of particles of the n -th generation for these processes is less than or equal to one.An unbiased estimator is constructed on the trajectory of the random process and ε - biased estimator on the process with a smaller number of branches is constructed.

The resulting estimates of the solution have a finite variance, is constructed on the trajectories of a branching process with a finite average number of branches. Using the apparatus of the theory of martingales and Markov times, we proved the unbiasedness and boundedness of the variance of the constructed estimators. At the end of paper based on the proposed estimators, computational experiments were carried out.

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A FREE BOUNDARY PROBLEM FOR A QUASILINEAR DIFFUSIVE WEAK COMPETITION MODEL

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This article is concerned free boundary problem for a system of parabolic equations, which arises in a competition ecological model:

$$\begin{cases} k_1(u)u_t - d_2u_{xx} - m_1u_x = u(a_1 - b_1u - c_1v), & (t,x) \in D, \\ k_2(v)v_t - d_2v_{xx} - m_2v_x = v(a_2 - b_2u - c_2v), & (t,x) \in D, \\ u(0,x) = u_0(x), & 0 \le x \le s_0 = s(0), \\ v(0,x) = v_0(x), & 0 \le x \le s_0, \\ u_x(t,0) = v_x(t,0) = 0, & 0 \le t \le T, \\ u(t,s(t)) = v(t,s(t)) = 0, & 0 \le t \le T, \\ s'(t) = -\mu\left(u_x(t,s(t)) + \rho v_x(t,s(t))\right), & 0 \le t \le T, \end{cases} \tag{1}$$

where $D = \{(t,x) : 0 < t \le T,\, 0 < x < s(t)\}$; $x = s(t)$ – free boundary, $u(t,x)$ and $v(t,x)$ represent densities of two competing species; $k_i \ge k_{0i}$, d_i, m_i, a_i, b_i, k_{0i}, c_i, μ, ρ – positive constants, $i = 1,2$. The initial functions $u_0(x)$, $v_0(x)$ satisfy

$$u_0(x), v_0(x) \in C^2[0,s_0],\ u_0(x) > 0 \text{ in } [0,s_0],\ v_0(x) > 0 \text{ in } [0,s_0],\ u_0(0) = v_0(0) = 0, u_0(s_0) = v_0(s_0) = 0,$$
$$u_0'(s_0) < 0,\ v_0'(s_0) < 0.$$

The model describes that two species $u(t,x)$ and $v(t,x)$ competing with each other in a one-dimensional habitat. We envision that the species initially occupy the region $[0,s_0]$ and have a tendency to expand their territory together.

The problem (1) has been studied in [1] for $k_i = 1$, $m_i = 0$. We extend some results of [1] and more general weak competition model ($\frac{c_1}{c_2} < \frac{a_1}{a_2} < \frac{b_1}{b_2}$).

For the solutions of (1) apriori estimates are established. On the base of apriori estimations the existence and uniqueness of theorems are proved. Also study asymptotic behavior of solutions.

Theorem 1. *Let $u\,(t,x)\,,v\,(t,x)\,,s\,(t)$ be a solution of* (1). *Then*

$$0 < u(t,x) \le M_1 \equiv \max\left\{\max_x \|u_0\|, \frac{a_1}{b_1}\right\}, \quad (t,x) \in \overline{D},$$

$$0 < v(t,x) \le M_2 \equiv \max\left\{\max_x \|v_0\|, \frac{a_2}{c_2}\right\}, \quad (t,x) \in \overline{D},$$

$$0 < s'(t) \le M_3 \equiv \mu\left(N_1 + \rho N_2\right), \quad 0 \le t \le T.$$

where $N_1 \ge \max\left\{a_1, \max\limits_x \frac{\|u_0\|}{s_0-x}\right\}$, $N_2 \ge \max\left\{\frac{a_2}{c_2}, \max\limits_x \frac{\|v_0\|}{s_0-x}\right\}$.

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QUANTUM FISHER INFORMATION AND WEHRL ENTROPY OF FIVE-LEVEL ATOM WITH SQUEEZED FIELD INTERACTION UNDER ENERGY DISSIPATION EFFECT

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Abstract. By using the analytical and numerical solution of the mathematical model of a five-level atom interacting with field initially in the squeezed state, we study the relationship among the quantum Fisher information, Wehrl entropy and Mandel parameter under energy dissipation effect. We illustrate the dynamical behavior of the three types of correlations in the absence and presence of energy dissipation. We find some monotonic correlation between the three correlations. Also, it is shown that the Wehrl entropy within the field basis is connected with the photon statistics and nonclassical properties of the field while the Wehrl entropy within the atomic basis is connected with the nonlocal correlation between the field and five-level atom during the interaction time. Furthermore, we discover that the three quantum quantifiers are strongly affected by system parameter such as the time-dependent coupling and squeeze parameter.

GLOBAL DYNAMICS OF THE REACTION-DIFFUSION SYSTEM SIMULATING THE SPREAD OF THE EPIDEMIC

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It is known that environmental change is one of the important factors affecting the emergence and spread of the epidemic. Due to environmental degradation, people are faced with many epidemics caused by environmental problems. Modern epidemics are characterized by high infectivity, rapid transmission, a wide range of epidemics, etc . For example, Feng et al. [1] studied a mathematical model of the epidemic with the environment as a transmission medium. We propose the following SIR reaction-diffusion model of an ecological epidemic in a heterogeneous space:

$$\begin{cases} \partial S(t,x) = \nabla \cdot D_1(S)\nabla S(t,x) + A(x) - \beta(x)Ef(S) - \mu(x)S, \\ \partial I(t,x) = \nabla \cdot D_2(I)\nabla I(t,x) + \beta(x)Ef(S) - (\mu(x) + \alpha(x) + \zeta(x))I, \\ \partial E(t,x) = \nabla \cdot D_3(E)\nabla E(t,x) + \theta(x)I(1-E) - (\xi(x) + \gamma(x))E, \end{cases} \quad (1)$$

$$S(0,x) = \phi_1(x), I(0,x) = \phi_2(x), E(0,x) = \phi_3(x), \quad \frac{\partial S(t,x)}{\partial n} = \frac{\partial I(t,x)}{\partial n} = \frac{\partial E(t,x)}{\partial n},$$

where $t \geq 0, x \in \Omega$, $\Omega \subset R^n$ is a bounded domain with a smooth boundary, and $n \geq 1$ is an integer, $S(t,x), I(t,x), R(t,x)$ and $E(t,x)$ is defined as the number of susceptible, infectious and distant people and the concentration of environmental pathogens (viruses or bacteria).

The study of the threshold dynamics of an epidemic with nonlinear morbidity is of great practical importance. Here, first of all, the correctness of decisions is achieved. Then, the base reproduction number R_0 is calculated. Finally, we establish a threshold condition for the global asymptotic stability of the painless equilibrium $W_0(x)$. From an epidemiological point of view, when $R_0 \leq 1$, the number of infected and the content of pathogens in the environment will gradually become zero, and eventually the epidemic will disappear. On the contrary, if $R_0 > 1$, we obtain uniform constancy of model (1). Moreover, when $R_0 > 1$, the global asymptotic stability of the endemic equilibrium $W* = (S*, I*, E*)$ is achieved in the spatially homogeneous case.

It can be seen that the elimination and prevalence of the epidemic are determined by the main reproductive number. The model proposed in this article can better characterize the environmental spread of epidemics and the impact of prevention and control measures on the spread of the epidemic.

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RESEARCH OF THE ISSUES OF CONVERGENCE AND STABILITY OF EXPLICIT AND IMPLICIT DIFFERENCE SCHEMES FOR SOLVING A NONLINEAR EQUATION WITH A FRACTIONAL DERIVATIVE OF VARIABLE ORDER

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The article discusses finite-difference schemes for the numerical solution of the fractional Riccati equation with variable coefficients and variable memory [1], where the fractional derivative is understood in the sense of Gerasimov-Caputo [2,3,4].

For a nonlinear fractional equation, in the general case, the theorems of approximation, stability and convergence of a non-local implicit finite-difference scheme [5] of the form are proved:

$$\frac{\tau^{-\alpha_k}}{\Gamma(2-\alpha_k)} \sum_{j=0}^{k-1} \left((j+1)^{1-\alpha_k} - j^{1-\alpha_k} \right) (u_{k-j} - u_{k-j-1}) - b_k u_k = f_k, \quad (1)$$
$$k = 1, ..., N, \qquad u_0 = const.$$

the scheme is shown conditionally converging with the first order of accuracy.

Assuming $f_k = -a_k u_k^2$ we pass to the numerical scheme for the fractional Riccati equation, which we solve the modified Newton method [5,6], for which it is shown that the method is locally: stable and convergent, with the first order of accuracy.

Also in the case of the fractional Riccati equation, theorems of approximation, stability and convergence are proved for a nonlocal explicit finite-difference scheme [7]. It is shown that the scheme converges conditionally with the first order of accuracy.

On specific test examples, the computational accuracy of numerical methods was estimated according to Runge's rule, and a comparison with the exact solution was made. It is shown that the order of the computational accuracy of numerical methods with increasing nodes of the computational grid tends to the theoretical order of accuracy.

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Section 3: Algebra and functional analysis

LOCAL DERIVATIONS ON NILPOTENT LEIBNIZ ALGEBRAS $NF_n \oplus F_m^1$

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In recent years non-associative analogues of classical constructions become of interest in connection with their applications in many branches of mathematics and physics. The notions of local and 2-local derivations become popular for some non-associative algebras such as the Lie and Leibniz algebras.

Investigation of local derivations on Lie algebras was initiated in paper in [3] of Sh.A.Ayupov and K.K.Kudaybergenov. They have proved that every local derivation on semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. In [4] local derivations and automorphism of complex finite-dimensional simple Leibniz algebras are investigated, and it is proved that all local derivations on finite-dimensional complex simple Leibniz algebras are automatically derivations; moreover, it is shown that filiform Leibniz algebras admit local derivations which are not derivations. The results of the papers [5] and [1] respectively, show that p-filiform Leibniz algebras and, respectively, quasi-filiform Leibniz algebras, as a rule admit local derivations which are not derivations.

In the present paper we describe local derivations on the Leibniz algebras of the form $NF_n \oplus F_1^m$ and show the existence of local derivations which are not derivations.

Definition 1. A vector space with a bilinear bracket $(L, [\cdot,\cdot])$ is called a Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity

$$\big[x, [y, z]\big] = \big[[x, y], z\big] - \big[[x, z], y\big],$$

holds.

For a given Leibniz algebra $(L, [\cdot,\cdot])$, the sequence of two-sided ideals are defined recursively as follows:

$$L^1 = L, \ \ L^{k+1} = [L^k, L], \ \ k \geq 1.$$

This sequence is said to be the lower central series of L.

Definition 2. A Leibniz algebra L is said to be nilpotent, if there exists $n \in \mathbb{N}$ such that $L^n = \{0\}$.

A derivation on a Leibniz algebra $\mathcal{L}$ is a linear map $D : \mathcal{L} \to \mathcal{L}$ which satisfies the Leibniz rule:

$$D([x, y]) = [D(x), y] + [x, D(y)], \quad \text{for any} \quad x, y \in \mathcal{L}. \tag{1}$$

The set of all derivations of a Leibniz algebra $\mathcal{L}$ is a Lie algebra with respect to commutation operation and it is denoted by $Der(\mathcal{L})$.

For any element $x \in \mathcal{L}$ the operator of right multiplication $ad_x : \mathcal{L} \to \mathcal{L}$, defined as $ad_x(z) = [z, x]$ is a derivation, and derivations of this form are called inner derivation. The set of all inner derivations of $\mathcal{L}$, denoted $ad(\mathcal{L})$, is an ideal in $Der(\mathcal{L})$.

Definition 3. A linear operator Δ is called a local derivation if for any $x \in \mathcal{L}$, there exists a derivation $D_x : \mathcal{L} \to \mathcal{L}$ (depending on x) such that $\Delta(x) = D_x(x)$. The set of all local derivations on $\mathcal{L}$ we denote by $LocDer(\mathcal{L})$.

Definition 4. An n-dimensional Leibniz algebra is called null-filiform if $\dim L^i = n + 1 - i, \quad 1 \leq i \leq n + 1$.

In [2] it is proved that up to isomorphism there exist a unique null-filiform Leibniz algebra and the multiplication of this algebra is the following (all remaining products of basis elements equal to zero):

$$NF_n : [e_i, e_1] = e_{i+1} \qquad 1 \leq i \leq n - 1.$$

Definition 5. A Leibniz algebra $\mathcal{L}$ is said to be filiform if $\dim L^i = n - i$, where $n = \dim \mathcal{L}$ and $2 \leq i \leq n$.

In [2] it is shown that up to isomorphism there exist two non-Lie naturally graded filiform Leibniz algebras

- $F_m^1 : [f_1, f_1] = f_3, \quad [f_i, f_1] = f_{i+1}, \quad 2 \le i \le m-1$
- $F_m^2 : [f_1, f_1] = f_3, \quad [f_i, f_1] = f_{i+1}, \quad 3 \le i \le m-1.$

Let us consider the direct sum of these algebras $\mathfrak{N} = NF_n \oplus F_m^1$. The following theorems describe derivations and local derivations of the Leibniz algebra $\mathfrak{N}$.

Theorem 1. A linear operator D on the Leibniz algebra $\mathfrak{N} = NF_n \oplus F_m^1$ is a derivation if and only if its matrix has the following form:

$$\begin{aligned}
D(e_1) &= \sum_{i=1}^{n} \alpha_i + \beta f_m,\\
D(e_j) &= j\alpha_1 e_j + \sum_{i=j+1}^{n} \alpha_{i-j+1} e_i, \quad 2 \le j \le n-1,\\
D(e_n) &= n\alpha_1 e_n,\\
D(f_1) &= \gamma e_n + \sum_{i=1}^{m} \lambda_i f_i,\\
D(f_2) &= \mu e_n + (\lambda_1 + \lambda_2) f_2 + \sum_{i=3}^{m-1} \lambda_i f_i + \sigma f_m,\\
D(f_j) &= ((j-1)\lambda_1 + \lambda_2) f_j + \sum_{i=j+1}^{m} \lambda_{i-j+2} f_i, \quad 3 \le j \le m.
\end{aligned}$$

Theorem 2. Let Δ be a linear operator on $\mathfrak{N}$. Then Δ is a local derivation, if and only if its matrix has the form:

$$\begin{aligned}
\Delta(e_1) &= \sum_{i=1}^{n} \alpha_{i,1} e_i + \alpha_{n+m,1} f_m,\\
\Delta(e_j) &= \sum_{i\ge j}^{n} \alpha_{i,j} e_i, \quad 2 \le j \le n,\\
\Delta(f_1) &= \alpha_{n,n+1} e_n + \sum_{i=1}^{m} \alpha_{n+i,n+1} f_i,\\
\Delta(f_2) &= \alpha_{n,n+2} e_n + \alpha_{n+2,n+2} f_2 + \sum_{i=3}^{m-1} \alpha_{n+i,n+1} f_i + \alpha_{n+m,n+2} f_m,\\
\Delta(f_j) &= \alpha_{n,n+1} e_n + \sum_{i\ge j}^{m} \alpha_{n+i,n+j} f_i, \quad 3 \le j \le m.
\end{aligned}$$

Remark. It easy to see that the dimensions of the space of derivations of algebra $\mathfrak{N}$ is equal to

$$\dim Der\mathfrak{N} = n + m + 4,$$

while the dimensions of the space of local derivations is equal to

$$\dim LocDer\mathfrak{N} = \frac{n^2 + n + m^2 - m + 12}{2}.$$

Therefore the dimension of the spaces of all local derivations of the Leibniz algebra $\mathfrak{N}$ is strictly greater than the dimension of the space of all derivations of $\mathfrak{N}$. Therefore, we have the following result.

Corollary. The Leibniz algebra $\mathfrak{N}$ admits local derivations which are not derivations.

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UNIFORMLY $R-$PARACOMPACT AND ITS HYPERSPACE

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Definition 1 [2]. Let X be a nonempty set. A family $\mathcal{U}$ of coverings of a set X is called uniformity on X if the following conditions are satisfied:

(1) If $\alpha \in \mathcal{U}$ and α is inscribed in some cover β of the set X, then $\beta \in \mathcal{U}$.

(2) For any $\alpha_1 \in \mathcal{U}$, $\alpha_2 \in \mathcal{U}$ there exists $\alpha \in \mathcal{U}$, which is inscribed in α_1 and α_2.

(3) For any $\alpha \in \mathcal{U}$, there exists $\beta \in \mathcal{U}$ strongly star inscribed in α.

(4) For any x, y of a pair of different points of X, there exists $\alpha \in \mathcal{U}$ such that no element of α contains both x and y.

A family $\mathcal{U}$ consisting of a set X, satisfying conditions (1) - (4) is called uniformity on X and a pair $(X, \mathcal{U})$ is called a uniform space.

For any uniformity of $\mathcal{U}$ on X, the family $\tau_{\mathcal{U}} = \{O \subset X : \text{for each}\, x \in O \,\text{exists}\, \alpha \in \mathcal{U} \,\text{such that}\, \alpha(x) \subset O\}$ is a topology on X and the topological space $(X, \tau_{\mathcal{U}})$ is a T_1-space. The topology of $\tau_{\mathcal{U}}$ is called the topology generated or induced by the uniformity of $\mathcal{U}$.

Let $(X, \mathcal{U})$ be a uniform space and $\exp X$ the set of all nonempty closed subsets of the space $(X, \tau_{\mathcal{U}})$. For each $\alpha \in \mathcal{U}$, put $P(\alpha) = \{\langle\alpha'\rangle : \alpha' \subseteq \alpha\}$, where

$$\langle\alpha'\rangle = \{F \in \exp X : F \subseteq \cup\alpha' \,\text{and}\, F \cap A \neq \varnothing \,\text{for each}\, A \in \alpha'\}.$$

Proposition 1 [2]. If $\mathcal{B}$ is the base of a uniform space $(X, \mathcal{U})$, then $P(\mathcal{B}) = \{P(\alpha) : \alpha \in \mathcal{B}\}$ forms a base of some uniformity $\exp\mathcal{U}$ on $\exp X$.

Remark 1 [2]. Let $\exp_c X$ be the set of all nonempty compact subsets of the uniform space $(X, \mathcal{U})$. For each $\alpha \in \mathcal{U}$, put $K(\alpha) = \{\langle\alpha'\rangle : \alpha' \subseteq \alpha \,\text{and}\, \alpha' - \text{finite}\}$. Note that $K(\alpha)$ is the cover of the set $\exp_c X$.

Corollary 1 [1]. If the uniform space $(X, \mathcal{U})$ is metrizable, then its hyperspace $(\exp X, \exp\mathcal{U})$ is also metrizable.

Let γ be the cover of the set X. Put $\gamma^{<} = \{\cup\gamma' : \gamma' - \text{finite subset of cover}\, \gamma\}$. The cover of γ is called additive if $\gamma^{<} > \gamma$.

A uniform space $(X, \mathcal{U})$ is called uniformly paracompact if, for any additive open cover γ of the space $(X, \mathcal{U})$, there exists a sequence $\{a_n\} \subset \mathcal{U}$ such that the following condition holds: $(\mathcal{U}\, P)$

For any point $x \in X$, there are a number $n \in N$ and an element $G \in \gamma$ such that $a_n(x) \subset \gamma$.

Theorem 1. A uniform space $(X, \mathcal{U})$ is uniformly paracompact if and only if a uniform space $(\exp_c X, \exp_c\mathcal{U})$ is uniformly paracompact.

A cover γ of a uniform space $(X, \mathcal{U})$ is said to be uniformly locally finite if there exists a uniform cover $\alpha \in \mathcal{U}$ such that each element A of α intersects only a finite number of elements of the cover γ.

Since the interiors of the elements of the uniform cover are a cover, then any uniformly locally finite cover is locally finite cover. A uniform space $(X, \mathcal{U})$ is said to be uniformly $R-$paracompact if any of its open cover can be inscribed with a uniformly locally finite open cover.

Theorem 2. A uniform space $(X, \mathcal{U})$ is uniformly $R-$paracompact if and only if the uniform space $(\exp_c X, \exp_c \mathcal{U})$ is uniformly $R-$paracompact.

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ACTING OF THE FUNCTOR OF G-PERMUTATION DEGREE ON UNIFORM CONTINUOUS MAPPING

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It is known that a *permutation group* is the group of all permutations, that is one-to-one mappings $X \to X$. A permutation group of a set X is usually denoted by $S(X)$. Especially, if $X = \{1, 2, \ldots, n\}$, then $S(X)$ is denoted by S_n.

Let X^n be the n-th power of a compact space X. The permutation group S_n of all permutations acts on the n-th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology is denoted by $SP^n X$. Thus, points of the space $SP^n X$ are finite subsets (equivalence classes) of the product X^n. Two points $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n) \in X^n$ are considered to be *equivalent* if there exists a permutation $\sigma \in S_n$ such that $y_i = x_{\sigma(i)}$ for each $i = 1, 2, \ldots, n$. The space $SP^n X$ is called the *n-permutation degree of the space* X. Equivalent relation by which we obtain space $SP^n X$ is called the *symmetric equivalence relation*. The n-th permutation degree is a quotient of X^n. Therefore, the quotient map is denoted by $\pi_n^s : X^n \to SP^n X$, where for every $x = (x_1, x_2, \ldots, x_n) \in X^n$,

$$\pi_n^s((x_1, x_2, \ldots, x_n)) = [(x_1, x_2, \ldots, x_n)]$$

is an orbit of the point $x = (x_1, x_2, \ldots, x_n) \in X^n$ [5].

Let G be a subgroup of the permutation group S_n and let X be a compact space. The group G acts also on the n-th power of the space X as permutation of coordinates. The set of all orbits of this action with quotient topology is denoted by $SP^n_G X$. The space $SP^n_G X$ is called *G-permutation degree* of the space X [4]. Actually, it is the quotient space of the product of X^n under the G-symmetric equivalence relation.

An operation SP^n is the covariant functor in the category of compacts and it is said to be a *functor of G-permutation degree.* If $G = S_n$, then $SP^n_G = SP^n$ and if the group G consists of the unique element only, then $SP^n = \Pi^n$.

Let T be a set and let A and B be subsets of $T \times T$, i.e., relations on the set T. The inverse relation of A will be denoted by A^{-1}, that is,

$$A^{-1} = \{(x, y) : (y, x) \in A\}.$$

The composition of A and B will be denoted by AB; thus we have

$$AB = \{(x, z) : \text{ there exists a } y \in T \text{ such that } (x, y) \in A \text{ and } (y, z) \in B\}.$$

For an arbitrary relation $A \subset T \times T$ and for a positive integer n the relation $A^n \subset T \times T$ is defined inductively by the formulas:

$$A^1 = A \text{ and } A^n = A^{n-1}A.$$

Every set $V \subset T \times T$ that contains the diagonal $\Delta_T = \{(x,x) : \ x \in T\}$ of T is called an *entourage of the diagonal.*

Definition 1. Let T be a non-empty set. A family $\mathcal{U}$ of subsets of $T \times T$ is called a *uniformity on* T, if this family satisfies following conditions:

(U1) Each $U \in \mathcal{U}$ contains the diagonal $\Delta_T = \{(x,\, x) : \, x \in T\}$ of T;

(U2) If $V_1,\, V_2 \in \mathcal{U}$, then $V_1 \cap V_2 \in \mathcal{U}$;

(U3) If $U \in \mathcal{U}$ and $U \subset V$, then $V \in \mathcal{U}$;

(U4) For each $U \in \mathcal{U}$ there is a $V \in \mathcal{U}$ with $V^2 \subset U$;

(U5) For each $U \in \mathcal{U}$ we have $U^{-1} \in \mathcal{U}$.

The pair $(T, \mathcal{U})$ is called *uniform space* [3]. Also, the elements of the uniformity $\mathcal{U}$ are called *entourages.*

For an entourage $U \in \mathcal{U}$ and a point $x \in T$ the set

$$U(x) = \{y \in T : \ (x, y) \in U\}$$

is called the U-*ball* centered at x. For a subset $A \subset T$ the set $U(A) = \bigcup\limits_{a \in A} U(a)$ is called the U-*neighborhood* of A [6].

Definition 2[1]. A function $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ is called uniformly continuous, if for each $V \in \mathcal{V}$ there exists a $U \in \mathcal{U}$ such that

$$(f \times f)(U) = \{(f(x_1),\, f(x_2)) : \, (x_1,\, x_2) \in U\} \subset V.$$

Definition 3[2]. The uniformly continuous function $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ is uniformly open, if for any $U \in \mathcal{U}$ there exists a $V \in \mathcal{V}$ such that

$$V(f(x)) \subset f(U(x))$$

for all $x \in X$.

Theorem 1. Let $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ be a uniformly continuous mapping. Then the mapping $SP^n_G f : \left(SP^n_G X,\, SP^n_G \mathcal{U}\right) \to \left(SP^n_G Y,\, SP^n_G \mathcal{V}\right)$ is also uniformly continuous.

Theorem 2. Let $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ be a uniformly open mapping. Then the mapping $SP^n_G f : \left(SP^n_G X,\, SP^n_G \mathcal{U}\right) \to \left(SP^n_G Y,\, SP^n_G \mathcal{V}\right)$ is also uniformly open.

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SPECTRA OF THE ENERGY OPERATOR OF ONE-ELECTRON SYSTEMS IN THE IMPURITY HUBBARD MODEL

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We consider the energy operator of one-electron system in impurity Hubbard model and investigate the spectra and local impurity states of this system in the two-dimensional case. The Hamiltonian of this system has the form $H = A\sum_{m,\gamma} a^{+}_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^{+}_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m} a^{+}_{m,\uparrow}a_{m,\uparrow}a^{+}_{m,\downarrow}a_{m,\downarrow} + (A_0 - A)\sum_{\gamma} a^{+}_{0,\gamma}a_{0,\gamma} + (B_0 - B)\sum_{\tau,\gamma}(a^{+}_{0,\gamma}a_{\tau,\gamma} + a^{+}_{\tau,\gamma}a_{0,\gamma}) + (U_0 - U)a^{+}_{0,\uparrow}a_{0,\uparrow}a^{+}_{0,\downarrow}a_{0,\downarrow}$. Here A (A_0) is the energy of the electron at the regular (impurity) lattice site, B (B_0) is the transfer integral between neighboring sites (between electron and impurities), $\tau = \pm e_j,\ j = 1, 2, ..., \nu$, where e_j are unit mutually orthogonal vectors (this means that summation is taken over the nearest neighbors), U (U_0) is the parameter of the on-site Coulomb interaction of two electrons in the regular (impurity) sites, γ is the spin index ($\gamma =\uparrow$ or $\gamma =\downarrow$, where $\uparrow$ and $\downarrow$ denote the spin values $\frac{1}{2}$ and $-\frac{1}{2}$), $a^{+}_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m \in Z^{\nu}$. The Hamiltonian H acts in the antisymmetric Fo'ck space $\mathcal{H}_{as}$. Let φ_0 be the vacuum vector in the space $\mathcal{H}_{as}$ and let $\widetilde{\mathcal{H}}_1$ be the subspace corresponding to the one-electron system, i.e, $\widetilde{\mathcal{H}}_1$ is the set of all vectors of the form $\psi_1 = \sum_{p\in Z^{\nu}} \widetilde{f}(p)a^{+}_{p,\uparrow}\varphi_0$. Denote by $\widetilde{H}_1$ the restriction of H to the subspace $\mathcal{H}_1$. Let $\nu = 2$.

Theorem 1. *a). If $\varepsilon_2 = -B$, and $\varepsilon_1 < -4B$ (respectively, $\varepsilon_2 = -B$, and $\varepsilon_1 > 4B$), then the operator $\widetilde{H}_1$ has a unique eigenvalue $z = A + \varepsilon_1$, lying the below (respectively, above) of the continuous spectrum $\sigma_{cont}(\widetilde{H}_1)$ of the operator $\widetilde{H}_1$.*

b). *If $\varepsilon_2 = -2B$ and $\varepsilon_1 < 0$ (respectively, $\varepsilon_2 = -2B$ and $\varepsilon_1 > 0$), then the operator $\widetilde{H}_1$ has a unique eigenvalue z (respectively, $\widetilde{z}$), lying the below (respectively, above) of $\sigma_{cont}(\widetilde{H}_1)$.*

c). *If $\varepsilon_2 = 0$ and $\varepsilon_1 < 0$ (respectively, $\varepsilon_2 = 0$ and $\varepsilon_1 < 0$), then the operator $\widetilde{H}_1$ has a unique eigenvalue z' (respectively, $\widetilde{z}'$), lying the below (respectively, above) of $\sigma_{cont}(\widetilde{H}_1)$.*

d). *If $\varepsilon_1 = 0$ and $\varepsilon_2 > 0$ (respectively, $\varepsilon_1 = 0$ and $\varepsilon_2 < -2B$), then the operator $\widetilde{H}_1$ has a unique eigenvalue z'' (respectively, $\widetilde{z}''$), lying the below (respectively, above) of $\sigma_{cont}(\widetilde{H}_1)$.*

e). *If $\varepsilon_1 = \frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_1 = -\frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$), then the operator $\widetilde{H}_1$ has a unique eigenvalue z_1 (respectively, $\widetilde{z}_1$), lying the below (respectively, above) of $\sigma_{cont}(\widetilde{H}_1)$.*

f). *If $\varepsilon_2 > 0$ and $\varepsilon_1 > \frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_2 < -2B$ and $\varepsilon_1 > \frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$), then the operator $\widetilde{H}_1$ has a unique eigenvalue z_2 (respectively, $\widetilde{z}_2$), lying the below (respectively, above) of $\sigma_{cont}(\widetilde{H}_1)$.*

k). *If $\varepsilon_2 > 0$ and $-\frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B} < \varepsilon_1 < \frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$ (respectively, $\varepsilon_2 < -2B$ and $-\frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B} < \varepsilon_1 < \frac{4(\varepsilon_2^2+2B\varepsilon_2)}{B}$), then the operator $\widetilde{H}_1$ has a exactly two eigenvalues z_1 and z_2 (respectively,$\widetilde{z}_1$ and $\widetilde{z}_2$), lying the below and above of $\sigma_{cont}(\widetilde{H}_1)$.*

m). *If $-2B < \varepsilon_2 < 0$, then the operator $\widetilde{H}_1$ has no eigenvalues lying the outside of the continuous spectrum of $\sigma_{cont}(\widetilde{H}_1)$.*

ON 5^{th}-DIMENSIONAL ALGEBRAS GENERATED BY QUADRATIC STOCHASTIC OPERATORS

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An algebra A over a field F is a vector space equipped with a bilinear product. The dimension $n = dim_F A$ of the space A over F is called the dimension of the algebra A over F. Below we consider real algebras, i.e., $F = R$. Let $\{a_1, a_2, ..., a_n\}$ be a basis of the real vector space A over R. For algebras over a field, the bilinear multiplication from $A \times A$ to A is completely determined by the multiplication of basis elements of A. Any algebra can be specified up to isomorphism by giving its dimension (say n), and specifying n^3 structure constants $c_{i,j,k} \in R$. These structure constants determine the multiplication in A via the following rule:

$$a_i a_j = \sum_{k=1}^{n} c_{ij,k} a_k.$$

For quadratic stochastic operator $V : S^{n-1} \to S^{n-1}$ with

$$(V_x)_k = \sum_{i,j=1}^{n} P_{ij,k} x_i x_j$$

algebra generated by this operator is defined by the following multiplication

$$a_i = \sum_{k=1}^{n} P_{ij,k} a_k.$$

We investigate algebras generated by extremal Volterra operators V on S^4 and prove the following statement.

Theorem For $n = 5$ an algebra A generated by extremal Volterra operator V will be associative if and only if the quadratic stochastic operator V is the regular transformation. The cases $n = 2$ and $n = 3$ have been studied in [1].

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SPECTRAL PROPERTIES OF SELF-ADJOINT PARTIALLY INTEGRAL OPERATORS WITH NON-DEGENERATE KERNELS

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Linear operators and equations with partial integrals in different functional spaces were studied in monograph [1]. The essential and discrete spectra [2] of partial integral operators (PIO) $T_1 + T_2$ of Fredholm type in the space $L_2([a, b] \times [c, d])$ with a degenerate kernel are studied in [3]. Let $\{\varphi_k(x)\}_{k=1}^{\infty}$ $\left(\{\psi_j(y)\}_{j=1}^{\infty}\right)$ be a complete orthonormal system of functions from the $L_2[a, b]$ $(L_2[c, d])$ and let $\{h_k(y)\}_{k=1}^{\infty}$ $\left(\{p_j(x)\}_{j=1}^{\infty}\right)$ be a system of essential bounded real functions [4] on $[c, d]$ (on $[a, b]$).

We define a measurable function $k_1(x, s, y)$ (function $k_2(x, t, y)$) in the Lebesgue sense on $[a, b]^2 \times [c, d]$ $\left(\text{on } [a, b] \times [c, d]^2\right)$ by the following rule:

$$k_1(x, s, y) = \sum_{k=1}^{\infty} \varphi_k(x)\overline{\varphi_k(s)} h_k(y) \quad \left(k_2(x, t, y) = \sum_{j=1}^{\infty} p_j(x)\psi_j(y)\overline{\psi_j(t)} \right),$$

where $|h_k(y)| \leq M_k$, $k \in \mathbb{N}$ and $\sum\limits_{k=1}^{\infty} M_k = M < \infty$ $(|p_j(x)| \leq N_j$, $j \in \mathbb{N}$ and $\sum\limits_{j=1}^{\infty} N_j = N < \infty)$.

We define T_1 (T_2) be a linear bounded self-adjoint PIO on the Hilbert space $L_2([a, b] \times [c, d])$ given by the formula

$$(T_1 f)(x,y) = \int\limits_a^b k(x,s,y) f(s,y) ds \quad \left((T_2 f)(x,y) = \int\limits_c^d k_2(x,t,y) f(x,t) dt \right).$$

Here integrals have to be understood in the Lebesgue sense on [a, b] (on [c, d]).

The spectrum, essential and discrete spectrum are denoted by σ, σ_{ess} and σ_{disc} respectively (see [2]).

Theorem. *For the essential spectrum $\sigma_{ess}(T_1+T_2)$ of the PIO T_1+T_2 with a non-degenerate kernels, the following formula*

$$\sigma_{ess}(T_1+T_2) = \{0\} \cup \left(\bigcup_{k=1}^{\infty} essran(h_k) \right) \cup \left(\bigcup_{j=1}^{\infty} essran(p_j) \right)$$

holds.

Also, a sufficient condition is obtained for the infinity of the discrete spectrum $\sigma_{disc}(T_1+T_2)$ PIO T_1+T_2.

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NATURALLY GRADED FILIFORM n-LIE ALGEBRA

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In this paper, we describe m-dimensional naturally graded filiform n-Lie algebras.

A vector space A over a field $\mathbb{F}$ is called an n-Lie algebra (Filippov algebra)[1] if an n-ary multilinear operation $[-,-,\dots,-]$, satisfying the following two identities:

$$[x_1, x_2, \dots, x_n] = (-1)^{sign(\sigma)}[x_{\sigma(1)}, x_{\sigma(2)}, \dots, x_{\sigma(n)}], \quad \sigma \in S_n$$

$$[[x_1,\dots,x_n], y_2, \dots, y_n] = \sum_{i=1}^{n} [x_1, \dots, x_{i-1}, [x_i, y_2, \dots, y_n], x_{i+1}, \dots, x_n].$$

Let A be a nilpotent n-Lie algebra. For the elements $\{x_1, x_2, \dots, x_{n-1}\}$ from $A \setminus A^2$, consider the ordered sequence $c(x_1, x_2, \dots, x_{n-1}) = (m_1, m_2, \dots, m_s)$, consisting of the sizes of the Jordan blocks of the operator $ad(x_1, x_2, \dots, x_{n-1})$.

The anticommutativity of the algebra A implies

$$c(x_1, x_2, \dots, x_{n-1}) = (m_1, m_2, \dots, m_k, \underbrace{1, 1, \dots, 1}_{n-1}),$$

where $m_1 \geq m_2 \geq \dots \geq m_k \geq 1$.

On the set of such sequences, we define the lexicographic order.

Definition 1.[3] Sequence

$$C(A) = \max_{x_i \in A \setminus A^2} \{c(x_1, x_2, \dots, x_{n-1})\}$$

is called the characteristic sequence of a finite-dimensional n- Lie algebra A.

Definition 2. An n-Lie algebra A of dimension m is called filiform if $dimA^i = m - n + 2 - i$ for $2 \leq i \leq m - n + 2$.

Let us characterize filiform n-Lie algebras in terms of the characteristic sequence.

Proposition 1. An n-Lie algebra of dimension m is filiform if and only if its characteristic sequence is $(m-n+1, \underbrace{1,1,\dots,1}_{n-1})$.

To describe m-dimensional naturally graded filiform n-Lie algebras, we need the following lemma.

Lemma 1. Let N be a nilpotent n-Lie algebra. Then for $i_1,\dots,i_k \in \mathbb{N},\ 2 \le k \le n$ the following embedding hold

$$[N^{i_1}, N^{i_2}, \dots, N^{i_k}, N, \dots, N] \subseteq N^{i_1+i_2+\dots+i_k-k+2}.$$

For a nilpotent algebra N we put

$$N_i := N^i/N^{i+1},\ 1 \le i \le s-1, \quad grN := N_1 \oplus N_2 \oplus \dots \oplus N_{s-1},$$

where $N^{s-1} \neq 0,\ N^s = 0$.

In the space grN we define the products as follows:

$$[\bar{e}_{i_1,p_1}, \bar{e}_{i_2,p_2}, \dots, \bar{e}_{i_n,p_n}]_{gr} = [e_{i_1,p_1}, e_{i_2,p_2}, \dots, e_{in,p_n}] + N^{i_1+i_2+\ dots+i_n-n+3},$$

where $\bar{e}_{i_j,p_j} = e_{i_j,p_j} + N^{i_j+1} \in N_{i_j},\ 1 \le j \le n$. By Lemma 1, these products are well defined. Moreover, it is fair to embed $[N_{i_1}, N_{i_2}, \dots, N_{i_n}] \subseteq N_{i_1+\dots+i_n-n+2}$, which implies that grN is graded n-Lie algebra.

Definition 3. If $grN \cong N$, then the n-Lie algebra N is called a naturally graded algebra.

Theorem 1. Let N be a naturally graded m-dimensional filiform n-Lie algebra. Then there exists a basis $\{e_1, e_2, \dots, e_m\}$ of N such that the multiplications table in this basis has the following form

$$\mathcal{NGF}_{m,n}(\beta_{i,j}): \begin{cases} [e_1, e_2, \dots, e_{n-1}, e_j] = e_{j+1}, \quad n \le j \le m-1, \\ [e_1, \dots, \widehat{e_i}, \dots, e_n, e_{n+j}] = \beta_{i,n+j} e_{n+j+1}, \quad 1 \le i \le n-1,\ 3 \le j \le m-n, \\ [e_1, \dots, \widehat{e_i}, \dots, \widehat{e_j}, \dots, e_n, e_{n+p}, e_{n+q}] = \gamma_{i,j}^{p,q} e_{n+p+q+1},\ 1 \le i < j \le n, \\ [e_1, \dots, \widehat{e}_{i_1}, \dots, \widehat{e}_{i_k}, \dots, e_n, e_{n+j_1}, \dots, e_{n+j_k}] = \gamma_{i_1,i_2,\dots,i_k}^{j_1,j_2,\dots,j_k} e_{n+j_1+j_2+\dots+j_k+1}, \\ [e_{n+j_1}, e_{n+j_2}, \dots, e_{n+j_n}] = \delta_{j_1,j_2,\dots,j_n} e_{n+j_1+j_2+\dots+j_n+1}, \end{cases}$$

where

$$\begin{cases} \gamma_{i,j}^{1,q} = \beta_{i,n+q}\beta_{j,n+q+1} - \beta_{j,n+q}\beta_{i,n+q+1}, & 1 \le i < j \le n-1, \quad 2 \le q \le m-n-2, \\ \gamma_{i,n}^{1,q} = \beta_{i,n+q} - \beta_{i,n+q+1}, & 1 \le i \le n-1, \quad 2 \le q \le m-n-2, \\ \gamma_{i,j}^{p,q} = \gamma_{i,j}^{p-1,q} + \beta_{i,n+q}\gamma_{j,n}^{p-1,q+1} - \beta_{j,n+q}\gamma_{i,n}^{p-1,q+1}, & 1 \le i < j \le n-1, \quad 1 \le p < q \le m-n-p-1, \\ \gamma_{i,n}^{p,q} = \gamma_{i,n}^{p-1,q} - \gamma_{i,n}^{p-1,q+1}, & 1 \le i \le n-1, \quad 1 \le p < q \le m-n-p-1, \end{cases}$$

$$\gamma_{i_1,i_2,\dots,i_k}^{j_1,j_2,\dots,j_k} = \sum_{t=1}^{k} (-1)^{k-t} \gamma_{i_1,\dots,\widehat{i}_t,\dots,i_k}^{j_2,\dots,j_k} \gamma_{i_t,n}^{n+j_1-1,n+j_2+\dots+j_k+1},$$

where $1 \le i_1 < i_2 < \dots < i_k \le n,\ 1 \le j_1 < j_2 < \dots < j_k \le m-n-1,\ 1 \le k \le n-1$,

$$\delta_{1,j_2,\dots,j_n} = \sum_{t=1}^{n-1} (-1)^{n+t} \gamma_{1,\dots,\widehat{t},\dots,n}^{j_2,\dots,j_n} \gamma_{t,n}^{n,n+j_2+\dots+j_n+1} + \gamma_{1,2,\dots,n-1}^{j_2,\dots,j_n},$$

where $2 \le j_2 < \dots < j_n \le m-n-j_2-\dots-j_{n-1}-2$,

$$\delta_{j_1,j_2,\dots,j_n} = \sum_{t=1}^{n-1} (-1)^{n+t} \gamma_{1,\dots,\widehat{t},\dots,n}^{j_2,\dots,j_n} \gamma_{t,n}^{n+j_1-1,n+j_2+\dots+j_n+1} + \delta_{j_1-1,j_2,\dots,j_n},$$

where $2 \le j_1 < j_2 < \dots < j_n \le m-n-j_1-j_2-\dots-j_{n-1}-1$.

An n-Lie algebra of the family $\mathcal{NGF}_{m,n}(\beta_{i,j})$ which $\beta_{i,j} = 0$ for any $1 \le i \le n-1$, $n+3 \le j \le m$, is denoted by $\mathcal{NGF}_{m,n}$. Hence,

$$\mathcal{NGF}_{m,n}: [e_1, e_2, \dots, e_{n-1}, e_j] = e_{j+1}, \quad n \le j \le m-1.$$

Note that if $n = 2$, then the algebra $\mathcal{NGF}_{m,n}$ coincides with the model filiform Lie algebra. The name of the model filiform Lie algebra is motivated by the fact that an arbitrary filiform Lie algebra is an infinitesimal deformation of the model filiform algebra. In the case $n > 3$, the algebra $\mathcal{NGF}_{m,n}$ is called filiform n-algebra [2].

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PERIODIC GIBBS MEASURES FOR THE HC MODEL WITH A COUNTABLE SET OF SPIN VALUES ON THE CAYLEY TREE

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Let $\Im^k = (V, L)$ is Cayley tree of order $k \geq 2$. Let $\Phi = \{\ldots, -1, 0, 1, \ldots\}$ and $\sigma \in \Omega = \Phi^V$ be a configuration. We consider the set Φ as the set of vertices of a graph G. A configuration σ is called a G-*admissible configuration* on the Cayley tree, if $\{\sigma(x), \sigma(y)\}$ is the edge of the graph G for any pair of nearest neighbors x, y in V.The activity set [1] for a graph G is a function $\lambda : G \to R_+$. For given G and λ we define the Hamiltonian of the $G-$HC model as $H_G^{\lambda}(\sigma) = \sum\limits_{x\in V} \log \lambda_{\sigma(x)}$, $\sigma \in \Omega^G$.

The reader can find the definition of the Gibbs measure and of other subjects related to Gibbs measure theory, for example, in [2].

We consider the case of the admissible configuration in the form: $\sigma(x)\sigma(y) = 0$ for any pair of nearest neighbors $l = \langle x, y\rangle$, $x, y \in V$. For any function $z : x \to z_x = (...,\ z_{-1,x}, z_{0,x},\ z_{1,x}, ...) \in R_+^{\infty}$ satisfying the equalities

$$z_{i,x} = \lambda_i \prod_{y\in S(x)} \frac{1}{1+\sum_{j\in\mathbb{Z}_0} z_{j,y}}, \quad i \in \mathbb{Z}_0 = \mathbb{Z}\backslash\{0\}, \lambda_i > 0. \tag{1}$$

there exists a unique HC-Gibbs measure μ, and vice versa. We study periodic solutions with a period of two. In the case from (1) for $G_k^{(2)}$-periodic Gibbs measures we have

$$\begin{cases} z_i = \lambda_i \cdot \left(\frac{1}{1+\sum_{j\in\mathbb{Z}_0} \widetilde{z}_j}\right)^k, \quad i \in \mathbb{Z}_0, \\ \widetilde{z}_i = \lambda_i \cdot \left(\frac{1}{1+\sum_{j\in\mathbb{Z}_0} z_j}\right)^k, \quad i \in \mathbb{Z}_0. \end{cases} \tag{2}$$

Analysis of the system of equations (2) gives the following theorem.

Theorem. Let $k \geq 2$ and $\Lambda_{cr} = \frac{k^k}{(k-1)^{k+1}}$. If the series $\sum_{j\in\mathbb{Z}_0} \lambda_j$ converges and its sum $\sum_{j\in\mathbb{Z}_0} \lambda_j = \Lambda$, then for HC model for $0 < \Lambda \leq \Lambda_{cr}$ there exist unique $G_k^{(2)}$-periodic Gibbs measure μ_0, wich is translation-invariant and for $\Lambda > \Lambda_{cr}$ there are exactly three $G_k^{(2)}$-periodic Gibbs measures μ_0, μ_1, μ_2, where measures μ_1 and μ_2 are $G_k^{(2)}$-periodic(not translation-invariant).

Remark. Measures μ_0, μ_1, μ_2 in the theorem are gradient Gibbs measures.

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CARATHEODORY NUMBERS CONNECTION WITH SIMPLEX

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This thesis about Caratheory numbers which is focusing in simplex. Below we write connection simplex with Caratheodory numbers such as nonlinear stochastic operators. This work is related quadratic operators.

Definition 1. The nonlinear stochastic operators are defined on a simplex S, and the dimensional of the simplex is $(m-1)$, as

$$S^{m-1} = \{x_i = (x_1, x_2, ..., x_m) \in R^m, \sum_{i=1}^{m} x_i = 1, x_i \geq 0\}$$

Definition 2. U is convex if $x, y \in U$ implies that $z = [\lambda x + (1-\lambda)y] \in U$ for all $\lambda \in [0,1]$.

A finite convexity space is a pair (V, C) where V is a finite set and C is a collection of subsets of V such that

1.$\emptyset, V \in C$, and

2.C is closed under intersecttions.

Definition 3. The Caratheodory number of (V, C) is the smallest integer k such that for every subset U of V and every element u in $H(U)$, there is a subset F of U with $|F| \leq k$ and $u \in H(F)$.

We well known caratheodory theorem: If $U \leq L$ (L-linear space) and its convex hull $H(U)$ has dimensions m, then for each $z \in H(U)$, there exists $m+1$ points $x_0, ..., x_m$ of U such that z is a convex combination of these points.

By the theorem we write lemma:

Lemma If a simplex consists of n points , then Caratheodory number is also n.

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EXPLICIT ESTIMATE FOR BLOW-UP SOLUTIONS OF NONLINEAR PARABOLIC SYSTEMS OF NON-DIVERGENCE FORM WITH VARIABLE DENSITY

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In this work, we study an estimate for blow-up solutions of nonlinear parabolic systems of non-divergence form with variable density

$$\begin{array}{l} |x|^{-l}\frac{\partial u}{\partial t} = v^{\alpha_1}\nabla\left(|x|^n u^{m_1-1}\nabla u\right) + |x|^{-l}u^{\beta_1}, \\ |x|^{-l}\frac{\partial v}{\partial t} = u^{\alpha_2}\nabla\left(|x|^n v^{m_2-1}\nabla u\right) + |x|^{-l}v^{\beta_2}, \end{array} \tag{1}$$

$$u|_{t=0} = u_0(x) \geq 0, \quad v|_{t=0} = v_0(x) \geq 0, \quad x \in R^N \tag{2}$$

where $\nabla(\cdot) = grad_x(\cdot)$, $m_1,\ m_2,\ \alpha_1,\ \alpha_2, \beta_1,\ \beta_2, n, l$ - are the positive numbers, $N \geq 1$ - the size of the space, $u = u(t,x) \geq 0,\ \ v = v(t,x) \geq 0$ - are the solutions.

Most of the known results in blow-up theory relate to processes described by second-order differential equations. A method for studying the blow-up situation for a wider class of problems based on the use of asymptotic a priori estimates was developed by S.I. Pohozhaev and E. Mitidieri [4, 5].

Using the method of comparison of solutions [1], without solving the problem, it is possible to estimate the solution from above and below [2], which is very important in the study of the properties of nonlinear problems [3]. In this work we obtain the estimates of the solution of problem (1), (2).

Then, this is validated by the following theorem:

Theorem. Let the following inequality $q_{3-i}\alpha_i + q_i(m_i - 1) + 1 > 0,\quad \beta_i < 1$ and $u_+(0,x) \geq u_0(x),\quad v_+(0,x) \geq v_0(x),\ \ x \in R^N$ be satisfied. Then the solution to problem (1) - (2) satisfies the following estimate

$$u(t,x) \leq u_+(t,x),\ v(t,x) \leq\ v_+(t,x)$$

where $u_+(t,x) = A_1(T-t)^{q_1}\left(a - \left(\frac{|x|}{(T-t)^{\frac{1}{2}}}\right)^2\right)^{\gamma_1}$, $v_+(t,x) = A_2(T-t)^{q_2}\left(a - \left(\frac{|x|}{(T-t)^{\frac{1}{2}}}\right)^2\right)^{\gamma_2}$

$\gamma_i, A_i,\ i = 1, 2$ - found constants.

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ON SKEW PRODUCTS OF QUADRATIC STOCHASTIC OPERATORS WITH n= m=2

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Let $V_1 : S^{(n-1)} \to S^{(n-1)}$ and $V_2 : S^{(m-1)} \to S^{(m-1)}$ - be quadratic stochastic operators, where $S^{(n-1)} = \{x = (x_1, .., x_n) : x_1 + .. + x_n = 1\}$; $S^{(m-1)} = \{y = (y_1, .., y_m) : y_1 + .. + y_m = 1\}$.

Let us define an operator

$V : S^{(n+m-1)} \to S^{(n+m-1)}$ by $(V(x,y))_i = (V_1(x))_i + g_i(x,y)$ for $i = 1, \ldots, n$

$(V(x,y))_{n+j} = (V_2(y))_j + g_{j+n}(x,y)$ for $j = 1, \ldots, m$

where $\sum\limits_{k=1}^{n+m} g_k = 2(x_1 + \ldots + x_n)(y_1 + \ldots + y_m)$.

If

$$g_i(x,y) = x_i(y_1 + \ldots + y_m)\ for\ i = 1, \ldots, n$$

$$g_{j+n}(x,y) = y_j(x_1 + \ldots + x_n)\ for\ \ j = 1, \ldots, m, \tag{1}$$

the corresponding operator is called the regular direct product of V_1 and V_2 and it is denoted by $V_1 \circledR V_2$. If $g_k(x,y)$ differ than (1), then corresponding operator is called a skew product.

In this paper for the case n=m=2 we consider quadratic stochastic operators

$$(V_1(x_1,x_2))_1 = x_1^2 + 2ax_1x_2\ ; \qquad (V_1(x_1,x_2))_1 = x_2^2 + 2(1-a)x_1x_2$$
$$(V_2(y_1,y_2)_2 = y_1^2 + 2by_1y_2; \qquad (V_2(y_1,y_2)_2 = y_2^2 + 2(1-b)y_1y_2$$

and its skew product

$$\begin{cases} (V_1 \otimes V_2)_1 = x_1^2 + 2ax_1x_2 + \alpha_1x_1x_3 + \alpha_2x_1x_4 \\ (V_1 \otimes V_2)_2 = x_2^2 + 2(1-a)x_1x_2 + \beta_1x_2x_3 + \beta_2x_2x_4 \\ (V_1 \otimes V_2)_3 = y_1^2 + 2by_1y_2 + \gamma_1x_1y_1 + \gamma_2y_1x_2 \\ (V_1 \otimes V_2)_4 = y_2^2 + (2-b)y_1y_2 + \delta_1y_2x_1 + \delta_2y_2x_2) \end{cases} \qquad (2)$$

We investigate a long run behavior of the trajectories of the skew product (2) for $a,b \in \{0,1\}, \alpha_1,\alpha_2,\beta_1,\beta_2 \in \{0,2\}$. It is established for which values of parameters the considered skew product V will be either ergodic or regular transformation. The n=2 and m=1 have been studied in [1].

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CARDINAL INVARIANTS OF HATTORI SPACES AND THEIR SUPEREXTENSIONS

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Let us recall (cf. [1]) that the Sorgenfrey line $(\mathbb{R},\tau_S)$ is the set $\mathbb{R}$ of real numbers with the lower limit topology τ_S (the family of all half-open intervals $[a,b)$, where $a<b$, is a base of the topology on $\mathbb{R}$). The space $(\mathbb{R},\tau_S)$ is an important example of topological spaces.

In [2] Hattori introduced a family $\mathfrak{G} = \{\tau(A) : A \subseteq \mathbb{R}\}$ of topologies on $\mathbb{R}$ such that $\tau(\mathbb{R}) = \tau_E$ (the Euclidean topology on $\mathbb{R}$) and $\tau(\emptyset) = \tau_S$. The topology $\tau(A)$ on $\mathbb{R}$ is defined as follows.

(1) For each $x \in A$, $\{(x-\varepsilon, x+\varepsilon) : \varepsilon > 0\}$ is the neighborhood base at x, and

(2) For each $x \in \mathbb{R} \setminus A$, $\{[x, x+\varepsilon) : \varepsilon > 0\}$ is the neighborhood base at x.

It is easy to see that for any $A, B \subseteq \mathbb{R}$ we have $A \supseteq B$ iff $\tau(A) \subseteq \tau(B)$. In particular, $\tau_E \subseteq \tau(A) \subseteq \tau_S$ for each $A \subseteq \mathbb{R}$.

The spaces $(\mathbb{R},\tau(A))$, $A \subset \mathbb{R}$, possess several nice topological properties (see [3]). In this note we will call the spaces $(\mathbb{R},\tau(A))$, $A \subset \mathbb{R}$, *Hattori spaces.*

It is well known that the set $D = \{(x,-x) : x \in \mathbb{R}\}$ of the space $(\mathbb{R},\tau(A))^2$ is closed and discrete. Moreover, $|\mathbb{R}^2| = \mathbf{c}$. Summarizing one get $e(\mathbb{R},\tau(A))^2 = s(\mathbb{R},\tau(A))^2 = \mathbf{c}$. It is also easy to see that $\pi w(\mathbb{R},\tau(A))^2 = c(\mathbb{R},\tau(A))^2 = \aleph_0$. We will generalize these facts.

Lemma 1. The space $\lambda_3(\mathbb{R},\tau(A))$ contains a closed discrete subset of cardinality $\mathbf{c}$.

Lemma 2. Let Y be a subset of a topological Hausdorff space X and $\mu_Y = \{Z : Z = \{F \in \mu \in \lambda_3 X : F \subset Y\}\}$. Denoted by $\lambda_\mu X$ the family of all systems μ_Y for all $Y \subset X$. It is easy to see that $\lambda_\mu X \subset \lambda_3 X$. Then

(i) the space $\lambda_3 Y$ is homeomorphic to the subspace $\lambda_\mu X$ of the space $\lambda_3 X$,

(ii) the set $\lambda_\mu X$ is open in $\lambda_3 X$ whenever Y is open in X,
(iii) the set $\lambda_\mu X$ is closed in $\lambda_3 X$ whenever Y is closed in X,
(iv) the set $\lambda_\mu X$ is clopen in $\lambda_3 X$ whenever Y is clopen in X.

Lemmas 1 and 2 implies

Proposition. Let A be subset of $\mathbb{R}$ and $B \subseteq \mathbb{R} \setminus A$. If B is a (resp. closed) subset of $(\mathbb{R}, \tau(A))$ which is homeomorphic to the space $(\mathbb{R}, \tau_S)$ then the space $\lambda_3(\mathbb{R}, \tau(A))$ contains a (resp. closed) discrete subset of cardinality $\mathbf{c}$.

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SOME TOPOLOGICAL PROPERTIES OF SPACE OF THE PERMUTATION DEGREE

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We say that a topological space X is a *Lindelof space*, if X is regular and every open cover of X has a countable subcover. A topological space X is said to be *locally Lindelof* if X is regular and every $x \in X$ has a neighbourhood which is Lindelof.

We observe that every Lindelof space is locally Lindelof but inverse is not true. For example, every uncountable discrete space is locally Lindelof but not Lindelof.

A permutation group X is the group of all permutations (i.s. one-one and onto mappings $X \to X$). A permutation group of a set X is usuallay denoted by $S(X)$. If $X = \{1, 2, ..., n\}$, then $S(X)$ is denoted by S_n, as well.

Let X^n be the n-th power of a compact X. The permutation group S_n of all permutations, acts on the n-th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by $SP^n X$. Thus, points of the space $SP^n X$ are finite subsets (equivalence classes) of the product X^n. Thus two points $(x_1, x_2, ..., x_n), (y_1, y_2, ..., y_n) \in X^n$ are considered to be equivalent if there is a permutation $\sigma \in S_n$ such that $y_i = x_(\sigma(i))$ for all $i = 1, 2, ..., n$. The space $SP^n X$ is called n-permutation degree of a space X. Equivalent relation by which we obtained space $SP^n X$ is called the symmetric equivalence relation. The n-th permutation degree is always a quotient of X^n. Thus, the quotient map is denoted by as following: $\pi_n^s : X^n \to SP^n X$, where for every $x = (x_1, x_2, ..., x_n) \in X^n$, $\pi_n^s((x_1, x_2, ..., x_n)) = [(x_1, x_2, ..., x_n)]$ is an orbit of the point $x = (x_1, x_2, ..., x_n) \in X^n$.

The concept of a permutation degree has generalizations. Let G be any subgroup of the group S_n. Then it also acts on X^n as group of permutations of coordinates. Consequently, it generates a G-symmetric equivalence relation on X^n. This quotient space of the product of X^n under the G-symmetric equivalence relation is called G-permutation degree of the space X and it is denoted by $SP^n_G X$. An operation SP^n_G is also the covariant functor in the category of compacts and it is said to be a functor of G-permutation degree. If $G = S_n$, then $SP^n_G = SP^n$. If the group G consists only of unique element, then $SP^n_G X = X^n$

Theorem.

a) If the product X^n is Lindelof space, then the space SP^nX is also Lindelof space;

b) If the product X^n is locally Lindelof space, then the space SP^nX is also locally Lindelof space.

Remark. Lindelofness and local Lindelofness are not preserved by even the finite products of the spaces. For example, the Sorgenfrey line K is a Lindelof space but the Sorgenfrey plane $K \times K$ is not even locally Lindelof.

Corollary. If the topological space X is Lindelof (resp. locally Lindelof), then in general, the space SP^nX is not always Lindelof (resp. locally Lindelof).

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GROUND STATES FOR THE q-STATE POTTS MODEL WITH AN EXTERNAL FIELD ON THE CAYLEY TREE

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Let $\tau^k = (V, L)$ be a Cayley tree of order k, i.e, an infinite tree such that exactly $k+1$ edges are incident to each vertex (see [1], [2]).

Let $\Phi = \{1, 2, 3, ..., q\}$. The set of all configurations coincides with the set $\Omega = \Phi^V$.

Potts model with an external field is given by Hamiltonian

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)} + \alpha \sum_{x\in V} \sigma(x),$$

where $J, \alpha \in \mathbb{R}$, $\alpha-$an external field and $\sigma \in \Omega$.

The energy of a configuration σ_b is defined by the formula

$$U(\sigma_b) = \frac{1}{2} J \sum_{x\in S_1(c_b)} \delta_{\sigma(x)\sigma(c_b)} + \alpha\sigma(c_b).$$

Lemma. For each σ_b configuration, we have the followings

$$U(\sigma_b) \in \{U_n : n = \overline{1, (k+2)q}\},$$

where $U_n = \left\{\frac{n-1}{k+2}\right\} \cdot \frac{k+2}{2} J + (1 + [\frac{n-1}{k+2}]) \cdot \alpha$.

Definition. A configuration φ is called a ground state for the Hamiltonian H, if

$$U(\varphi_b) = \min\{U_n : n = \overline{1, (k+2)q}\},$$

for any $b \in M$.

For every $m = \overline{1, (k+2)q}$, we put $A_m = \{(J, \alpha) : U_m = \min\{U_n, n = \overline{1, (k+2)q}\}\}$.
Using from this, we obtain the following sets
$A_{k+2} = \{(J, \alpha) \in \mathbb{R}^2 : J \leq 0, \alpha \geq 0\}$, $A_{(k+2)q} = \{(J, \alpha) \in \mathbb{R}^2 : J \leq 0, \alpha \leq 0\}$.

The following theorem describes the set of all ground states of the Hamiltonian H $(GS(H))$.

Theorem. Let $\alpha \neq 0$. The following assertions hold

1) if $(J, \alpha) \in A_{k+2}$ then $GS(H) = \{\sigma(x) = 1, \forall x \in V\}$;
2) if $(J, \alpha) \in A_{(k+2)q}$ then $GS(H) = \{\sigma(x) = q, \forall x \in V\}$;
3) if $(J, \alpha) \in \mathbb{R}^2 \setminus (A_{k+2} \cup A_{(k+2)q})$ then $GS(H) = \emptyset$.

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ON TRAJECTORIES OF A QUADRATIC OPERATOR ON THE TWO-DIMENSIONAL SIMPLEX

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Let $X \in \mathbb{R}^d$ be a set and $W : X \to X$ be a mapping.

For arbitrary given vector $x_0 \in X$ the discrete-time dynamical system of x_0 under operator W is the sequence of vectors of X

$$x_0,\ x_1 = W(x_0),\ x_2 = W^2(x_0),\ x_3 = W^3(x_0),\ \dots \tag{1}$$

where $W^n(x)$ is the n-fold composition of W with itself.

The main problem: is to know what ultimately happens with the sequence (1). Does the limit $\lim_{n\to\infty} x_n$ exist? This is very difficult problem, in general (see [1],[2]).

Let $X = S^2 = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \geq 0, \quad x + y + z = 1\}$ be two-dimensional simplex. Consider the following quadratic non-stochastic operator $W : (x, y, z) \in S^2 \to (x', y', z') = W(x, y, z) \in S^2$:

$$W : \quad x' = x^2 + y^2 + z^2 - axy - axz + 2yz, \quad y' = (2 + a)xy, \quad z' = (2 + a)xz, \tag{2}$$

where $a \in [0, 2]$.

The fixed points of this operator are solutions to the system $W(x, y, z) = (x, y, z)$.

In case $y = z = 0$ we get the fixed point $p_0 = (1, 0, 0)$. For $y + z \neq 0$ we get $x = \frac{1}{2+a}$. Consequently, the following is a family of fixed points

$$p_y = \left(\frac{1}{2+a}, y, 1 - \frac{1}{2+a} - y\right), \quad \text{where} \quad y \in \left[0, 1 - \frac{1}{2+a}\right].$$

A set M is called invariant with respect to an operator V if $V(M) \subset M$.

It is easy to see that the following sets are invariant with respect to (2):

$$M_0 = \{(x, y, z) \in S^2 : y = 0\}, \quad M_1 = \{(x, y, z) \in S^2 : z = 0\},$$
$$M_\omega = \{(x, y, z) \in S^2 : y = \omega z\}, \quad \omega \in [0, +\infty).$$

The simplex S^2 is partitioned by invariant sets of W as

$$S^2 = M_0 \cup M_1 \cup \Big(\bigcup_{\omega\in(0,+\infty)} M_\omega\Big).$$

Therefore, it suffices to study restrictions of the operator (2) on each invariant sets.

Theorem. 1) The restrictions of the operator on the invariants M_0, M_1 and M_ω are given by the same logistic function.

2) If $1.56995 < a \leq 2$ then each restricted one-dimensional dynamical system is chaotic.

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DYNAMICS OF A NON-STOCHASTIC QUADRATIC OPERATOR

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Let $S^2 = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \geq 0, \quad x + y + z = 1\}$ be two-dimensional simplex. Consider the following quadratic non-stochastic operator $W : (x, y, z) \in S^2 \to (x', y', z') = W(x, y, z) \in S^2$:

$$W : x' = x^2 + y^2 + z^2 - axy + 2xz - yz, \quad y' = (2 + a)xy, \quad z' = 3yz, \tag{2}$$

where $a \in [0, 2]$.

For arbitrary given vector $x_0 \in S^2$ the discrete-time dynamical system of x_0 under operator W is the sequence of vectors of S^2

$$x_0,\, x_1 = W(x_0),\, x_2 = W^2(x_0),\, x_3 = W^3(x_0),\, \ldots \tag{1}$$

where $W^n(x)$ is the n-fold composition of W with itself.

The main problem: is to know what ultimately happens with the sequence (1). Does the limit $\lim_{n\to\infty} x_n$ exist? This is very difficult problem, in general (see [1],[2]).

The fixed points of this operator are solutions to the system $W(x, y, z) = (x, y, z)$.

By simple calculations we get the set of all fixed points of operator (2):

$$\mathrm{Fix}(W) = \left\{ A(1, 0, 0), B\left(\frac{1}{2+a}, \frac{1+a}{2+a}, 0\right), C\left(\frac{1}{2+a}, \frac{1}{3}, \frac{2a+1}{3(2+a)}\right)\right\}.$$

Definition. *A fixed point x^* of a mapping F is called* **hyperbolic** *point if its Jacobian J_F at x^* has no eigenvalues on the unit circle;* **attracting** *point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are less than 1 in absolute value;* **repelling** *point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are greater than 1 in absolute value; a* **saddle** *point otherwise.*

The following theorem gives type (see [1] for definitions) of each fixed point of the operator W:

Theorem 1. 1) *The fixed point A is a saddle point for each $a \in [0, 2]$.*

2) The fixed point B is saddle if $a \in [0, 1)$; non-hyperbolic for $a = 1$ and repelling point if $a \in (1, 2]$.

3) The fixed point C is attractive if $a \in [0, 1)$; non-hyperbolic for $a = 1$ and repelling point if $a \in (1, 2]$.

Theorem 2. *If $a \in [0, 1)$ then there is an open neighborhood U_C of the fixed point C such that*

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} W^n(x_0) = C, \quad \forall x_0 \in U_C$$

We note that for $a \in [1, 2]$ the behavior of (1) is not clear.

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STRUCTURE OF ESSENTIAL SPECTRA AND DISCRETE SPECTRUM OF THE ENERGY OPERATOR OF SIX ELECTRON SYSTEMS IN THE HUBBARD MODEL. SECOND SINGLET STATE

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We consider of the energy operator of six electron systems in the Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system in the second singlet state. Hamiltonian of considering system has the form $H = A\sum_{m,\gamma} a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma}a_{m+\tau,\gamma} +$

$U\sum_m a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow}$. Here, A is the electron energy at a lattice site, B is the transfer integral between neighboring sites (we assume that $B>0$ for convenience), τ which means that summation is taken over the nearest neighbors, U is the parameter of the on-site Coulomb interaction of two electrons, γ is the spin index, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m\in Z^\nu$. In the six-electron systems exists singlet and triplet and quintet and octet states. The Hamiltonian H acts in the antisymmetric Fo'ck space $\mathcal{H}_{as}$. Let φ_0 be the vacuum vector in the space $\mathcal{H}_{as}$. The second singlet state corresponds the basis functions ${}^2s^0_{p,q,r,t,k,n}=a^+_{p,\uparrow}a^+_{q,\downarrow}a^+_{r,\uparrow}a^+_{t,\downarrow}a^+_{k,\uparrow}a^+_{n,\downarrow}\varphi_0$. The subspace ${}^2\widetilde{\mathcal{H}}^s_0$, corresponding to the second singlet state is the set of all vectors of the form ${}^2\psi^s_0=\sum_{p,q,r,t,k,n\in Z^\nu}\widetilde{f}(p,q,r,t,k,n)^2s^0_{p,q,r,t,k,n}$, $\widetilde{f}\in l^{as}_2$, where l^{as}_2 is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^6)$. Let $\nu=3$, $\Lambda_1=\lambda+\mu$, $\Lambda_2=\gamma+\theta$, $\Lambda_3=\eta+\xi$, and $\Lambda_i=(\Lambda^0_i,\Lambda^0_i,\Lambda^0_i)$, $i=1,2,3$;

Theorem 1. *a). If $U<0$, and $U<-\frac{12B\frac{\Lambda^0_3}{2}}{W}$, and $\cos\frac{\Lambda^0_1}{2}<\cos\frac{\Lambda^0_2}{2}$, $\cos\frac{\Lambda^0_2}{2}<3\cos\frac{\Lambda^0_3}{2}$, then the essential spectrum of operator ${}^2\widetilde{H}^s_0$ is consists of the union of seven segments: $\sigma_{ess}({}^2H^s_0)=[a_1+c_1+e_1,b_1+d_1+f_1]\cup[a_1+c_1+z_3,b_1+d_1+z_3]\cup[a_1+e_1+z_2,b_1+f_1+z_2]\cup[a_1+z_2+z_3,b_1+z_2+z_3]\cup[c_1+e_1+z_1,d_1+f_1+z_1]\cup[c_1+z_1+z_3,d_1+z_1+z_3]\cup[e_1+z_1+z_2,f_1+z_1+z_2]$, and discrete spectrum of operator ${}^2\widetilde{H}^s_0$ of no more one eigenvalue: $\sigma_{disc}({}^2H^s_0)=\{z_1+z_2+z_3\}$, or $\sigma_{disc}({}^2H^s_0)=\emptyset$. Here, and hereafter $a_1=2A-12B\cos\frac{\Lambda^0_1}{2}$, $b_1=2A+12B\cos\frac{\Lambda^0_1}{2}$, $c_1=2A-12B\cos\frac{\Lambda^0_2}{2}$, $d_1=2A+12B\cos\frac{\Lambda^0_2}{2}$, $e_1=2A-12B\cos\frac{\Lambda^0_3}{2}$, $f_1=2A+12B\cos\frac{\Lambda^0_3}{2}$, and z_1,z_2, and z_3 are the certain number's.*

b). *If $-\frac{12B\cos\frac{\Lambda^0_3}{2}}{W}\le U<-\frac{4B\cos\frac{\Lambda^0_2}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2}<\cos\frac{\Lambda^0_2}{2}$, and $\cos\frac{\Lambda^0_2}{2}<3\cos\frac{\Lambda^0_3}{2}$, then the essential spectrum of the operator ${}^2\widetilde{H}^s_0$ is consists of the union of four segments: $\sigma_{ess}({}^2H^s_0)=[a_1+c_1+e_1,b_1+d_1+f_1]\cup[a_1+e_1+z_2,b_1+f_1+z_2]\cup[c_1+e_1+z_1,d_1+f_1+z_1]\cup[c_1+z_1+z_2,d_1+z_1+z_2]$, and the discrete spectrum of the operator ${}^2\widetilde{H}^s_0$ is empty.*

c).*If $-\frac{4B\cos\frac{\Lambda^0_2}{2}}{W}\le U<-\frac{12B\cos\frac{\Lambda^0_3}{2}}{W}$, $\cos\frac{\Lambda^0_1}{2}>\cos\frac{\Lambda^0_2}{2}$, and $\cos\frac{\Lambda^0_2}{2}>3\cos\frac{\Lambda^0_3}{2}$, then the essential spectrum of operator the operator ${}^2\widetilde{H}^s_0$ is the union of two segments: $\sigma_{ess}({}^2H^s_0)=[a_1+c_1+e_1,b_1+d_1+f_1]\cup[a_1+c_1+z_3,b_1+d_1+z_3]$, and discrete spectrum of the operator ${}^2\widetilde{H}^s_0$ is empty set.*

d). *If $-\frac{12B\cos\frac{\Lambda^0_3}{2}}{W}\le U<0$, $\cos\frac{\Lambda^0_1}{2}>\cos\frac{\Lambda^0_2}{2}$ and $\cos\frac{\Lambda^0_2}{2}>3\cos\frac{\Lambda^0_3}{2}$, then the essential spectrum of the operator ${}^2\widetilde{H}^s_0$ is consists of a single segment: $\sigma_{ess}({}^2H^s_0)=[a_1+c_1+e_1,b_1+d_1+f_1]$, and discrete spectrum of the operator ${}^2\widetilde{H}^s_0$ is empty set: $\sigma_{disc}({}^2\widetilde{H}^s_0)=\emptyset$.*

LOCAL AUTOMORPHISM ON SOME NILPOTENT LEIBNIZ ALGEBRAS

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Investigation of local derivations on Lie algebras was initiated in paper in [1]. Sh.A.Ayupov and K.K.Kudaybergenov have proved that every local derivation on semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. In [2] local derivations and automorphism of complex finite-dimensional simple Leibniz algebras are investigated, and it is proved that all local derivations on a finite-dimensional complex simple Leibniz algebras are automatically derivations and it is shown that filiform Leibniz algebras admit local derivations which are not derivations. In the present paper we describe local automorphism of $NF_k\oplus NF_s$ Leibniz algebra and show the existence of local automorphism on $NF_k\oplus NF_s$ Leibniz algebra which is not automorphism.

Definition 1. A vector space with a bilinear bracket $(L, [\cdot,\cdot])$ is called a Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity

$$\big[x,[y,z]\big] = \big[[x,y],z\big] - \big[[x,z],y\big],$$

holds.

A linear bijective map $\varphi : \mathcal{L} \to \mathcal{L}$ is called an automorphism (resp. an anti- automorphism), if it satisfies $\varphi([x,y]) = [\varphi(x),\varphi(y)]$(resp. $\varphi([x,y]) = [\varphi(y),\varphi(x)]$) for all $x, y \in \mathcal{L}$.

Definition 2. Let $\mathcal{L}$ be an algebra. A linear map $\Phi : \mathcal{L} \to \mathcal{L}$ is called a local automorphism, if for any element $x \in A$ there exists an automorphism $\varphi_x : \mathcal{L} \to \mathcal{L}$ such that $\Phi(x) = \varphi_x(x)$.

An n-dimensional Leibniz algebra is called null-filiform if $\dim L^i = n + 1 - i, \quad 1 \le i \le n+1$.

Let NF_k be an k-dimensional null-filiform Leibniz algebra with a basis $e_1, e_2, ..., e_k$ and NF_s an s-dimensional null-filiform Leibniz algebra with a basis $f_1, f_2, ..., f_s$ then we have the following multiplication[3]:

$$NF_k : \ [e_i, e_1] = e_{i+1}, \quad 1 \le i \le k-1; \quad NF_s : \ [f_i, f_1] = f_{i+1}, \quad 1 \le i \le s-1.$$

Let us consider the direct sum of these algebras $\mathfrak{L} = NF_k \oplus NF_s$. The following proposition describes automorphisms of the algebra $\mathfrak{L}$.

Proposition 1. Any automorphisms of the algebra $\mathfrak{L} = NF_k \oplus NF_s$ has the following matrix form:

$$\begin{aligned} &\varphi(e_1) = \sum_{i=1}^{k} \alpha_i e_i + \lambda_s f_s, \quad \varphi(e_j) = \alpha_1^{j-1} \sum_{i=1}^{k+1-j} \alpha_i e_{i+j-1}, \ 2 \le j \le k, \\ &\varphi(f_1) = \mu_k e_k + \sum_{i=1}^{s} \beta_i f_i, \quad \varphi(f_j) = \beta_1^{j-1} \sum_{i=1}^{s+1-j} \beta_i f_{i+j-1}, \ 2 \le j \le s, \end{aligned} \tag{1}$$

where $\alpha_i, \ \beta_i, \ \lambda_i, \ \mu_i \in \mathbb{C}, \ \alpha_1 \neq 0, \ \beta_1 \neq 0$.

Theorem 1. The algebra $\mathfrak{L}$ admit local automorphism which are not automorphism.

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Section 4: Theoretical and Mathematical physics, ill-posed and inverse problems

ABOUT THE SOLUTIONS OF THE SYSTEMS OF LINEAR FREDHOLM INTEGRAL EQUATIONS OF THE THIRD KIND WITH MULTIPOINT SINGULARITIES IN THE SEMIAXIS

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Consider the system of linear integral equations of the third kind

$$p_i(x)u_i(x) = \lambda \sum_{j=1}^{n} \int_a^\infty k_{ij}(x,y)u_j(y)dy + f_i(x), x \in [a,\infty), \tag{1}$$

where $i = 1, 2, ...n, p_i(x)$ and $f_i(x)$ are given continuous functions on $[a,\infty)$, $k_{ij}(x,y)$ are given continuous functions in $G = [a,\infty) \times [a,\infty)$, $u_i(x)$ are the sought functions on $[a,\infty)$, $i, j = 1, 2, ..., n$; and λ is a real parameter. There exists $t \in \{1, 2, ..., n\}$ such that, for all $i = t, t+1, ..., n$ and $l = 1, 2, ..., m(i), p_i(x_{il}) = 0$, where $x_{il} \in [a,\infty)$ and for all $i = 1, 2, ..., t-1, p_i(x) = 1$ for all $x \in [a,\infty)$.

Various issues concerning the theory of integral equations were studied in [1-7]. Specifically, in [5] Lavrent'ev constructed regularizing operators for solving linear Fredholm integral equations of the first kind. In [6] uniqueness theorems were proved for systems of linear Fredholm integral equations of the third kind and regularizing operators in the sense of Lavrent'ev were constructed. In [7] a new approach was used to analyze the existence and uniqueness of solutions to the systems of linear Fredholm integral equations of the third kind on a finite interval with finitely many multipoint singularities.

Here, a new approach, we prove that the solution of system (1) in the space $L_{2,n}(a,\infty)$ is equivalent to the solution of systems of linear integral equations of the second kind with the some integral conditions. **References**

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THE SOLVABILITY OF MIXED PROBLEM FOR A LOADED PARABOLIC-HYPERBOLIC EQUATION

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Let D be a domain bounded by segments $y=0$, $x=1$, $y=1$ and $x=-1$ and we introduce the following notations

$$D_1 = D\cap\{x>0\},\ \ D_2 = D\cap\{x<0\},\ \ I=\{(x,y): x=0, 0<y<1\}.$$

We consider the following linear loaded integro-differential equation

$$u_{xx}-\frac{1-sgny}{2}u_{yy}-\frac{1+sgny}{2}u_y+c_i(x,y)u+Mu(\Theta(x),0)=0,\ \text{ in }\ D_i, \tag{1}$$

where $Mu(\Theta(x),0)=\sum\limits_{k=1}^{n}a_k(x,y)D_{oy}^{\alpha_k}u(x,0)$ in D_1, $Mu(\Theta(x),0)=\sum\limits_{k=1}^{n}b_k(x,y)D_{oy}^{\beta_k}u(0,y)$, in D_2, $D_{ax}^{\alpha_k}$ ($D_{a\xi}^{\beta_k}$) are the the Riemann-Liouville fractional integral operator of order $\alpha_k(\beta_k)<0$. We assume that the functions $a_k(x,y)$, $b_k(x,y)$ and $c_i(x,y)(i=1,2)$ are continuously differentiable with respect to Holder into the closure of domain D.

In this work, we will investigate the mixed problem with the integral gluing conditions for a loaded integro-differential equation of parabolic-hyperbolic type.

Problem 1. Find a function $u(x,y)$ satisfying the conditions:

1) $u(x,y)\in C(\bar{D}_i)\cap C'(D_i\cup I)$;

2) $u(x,y$ is a regular solution of (1) in the domains $D_i(i=1,2)$;

3) the following gluing conditions

$$\left.\begin{aligned}&\tau_1(y)=\mu(y)\tau_2(y)+\sigma(y),\\ &\nu_1(y)=\int\limits_0^y\gamma(y,\eta)\nu_2(\eta)d\eta+\delta(y)\nu_2(y)+\xi(y)\tau_2(y)+\theta(y),\end{aligned}\right\} \tag{2}$$

are satisfied on I, $\tau_1(y)=u(+0,y)$, $\nu_1(y)=u_x(+0,y)$, $\tau_2(y)=u(-0,y)$, $\nu_2(y)=u_x(-0,y)$;

4) satisfies boundary conditions:

$$u(-1,y)=\varphi_1(y),\quad u(1,y)=\varphi_2(y),\quad 0\le y\le 1,$$
$$u(x,0)=\psi_1(x),\quad u_y(x,0)=0,\quad -1\le x\le 0,$$
$$u(x,0)=\psi_2(x),\quad 0\le x\le 1,$$

here $\psi_1''(x)$, $\varphi_1''(y)$, $\psi_2''(x)$, $\varphi_2(y)$, $c_1(x,y)$, $\mu'(y)$, $\sigma'(y)$, $\delta'(y)$, $\gamma_y(y,\eta)$, $\xi'(y)$, $\theta'(y)$-satisfy the Holder condition, and $c_1(x,y)\le 0$, $\mu(y)\ne 0$, $\gamma^2(y,\eta)+\delta^2(y)\ne 0$; moreover,

$$\varphi_1(0)=\psi_1(1),\quad \varphi_2(y)=\psi_2(1),\quad \psi_2(0)=\mu(0)\psi_1(0)+\sigma(0).$$

Considered problem equivalently reduced to the loaded integral equation. We note that problem 1 arises when studying the propagation of an oscillation in composite electric lines and jointly mixed motions of viscous and viscoelastic fluids in a flat tube.

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DIRECT DERIVATION OF THE GROSS-PITAEVSKII EQUATION FROM THE BBGKY CHAIN OF QUANTUM KINETIC EQUATIONS

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A new method is proposed to obtain Gross-Pitaevskii equation by the chain of Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) quantum kinetic equations. In that sense, we investigate the dynamics of a quantum system including infinite number of identical particles which interact via a (special) pair potential on the form of Dirac delta-function.

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FLUORESCENCE SPECTRUM OF A LASER DRIVEN POLAR QUANTUM EMITTER DAMPED BY DEGENERATE SQUEEZED VACUUM WITH FINITE BANDWIDTH

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A two-level quantum emitter with broken inversion symmetry driven by external semiclassical monochromatic high-frequency electromagnetic (e.g. laser) field and damped by squeezed vacuum reservoir with finite bandwidth was studied. The squeezed vacuum source is assumed to be a degenerate parametric oscillator (DPO). It is shown that the shape of low-frequency fluorescence spectrum of the emitter can be effectively alternated by controlling the degree of the vacuum source squeezing and phase of the squeezing.

AN ILL-POSED BOUNDARY VALUE PROBLEM FOR THE MIXED TYPE EQUATION IN THE MULTIDIMENSIONAL CASE

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Let $Q = \{(x,t) : x \in \Omega, t \in (0,T)\}$, $x = (x_1, x_2, ..., x_n)$, $\Omega = \{-l < x_1 < l, 0 < x_i < d_i, i = 2, 3, ...n\}$, S - boundary of the region Ω, $\bar{\Omega} = \Omega \cup S$.

In the region $Q \cap \{x_1 \neq 0\}$ we consider the differential equation

$$u_{tt} + a_1 \operatorname{sgn}(x_1)\, u_{x_1x_1} + \sum_{i=2}^{n} a_i u_{x_ix_i} + \sum_{i=1}^{n} b_i u_{x_i} + cu = f, \tag{1}$$

where a_i, b_i, c - some constants, $i = 1, 2, ..., n$, $f(x,t)$ - given function.

Formulation of the problem. Find a function $u(x,t)$ satisfying equation (1) in the region $Q \cap \{x_1 \neq 0\}$ and the following conditions:
initial

$$u|_{t=0} = \varphi(x), \;\; u_t|_{t=0} = \psi(x), x \in \bar{\Omega}, \tag{2}$$

boundary

$$\begin{array}{l} u|_{x_1=-l} = u|_{x_1=l} = 0, \\ u|_{x_i=0} = u|_{x_i=d_i} = 0, \;\; i = 2, ..., n., \end{array} \tag{3}$$

and gluing conditions

$$u|_{x_1=-0} = u|_{x_1=+0}, \;\; u_{x_1}|_{x_1=-0} = u_{x_1}|_{x_1=+0}, \tag{4}$$

where $\varphi(x)$, $\psi(x)$ are sufficient smooth functions and satisfy the consistency conditions.

In this paper, we obtain an a priori estimate for the solution of the problem (1) - (4) and prove uniqueness and conditional stability theorems on the well-posedness set.

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REGULARIZATION OF THE ILL-POSED CAUCHY PROBLEM FOR MATRIX FACTORIZATIONS OF THE HELMHOLTZ EQUATION IN A MULTIDIMENSIONAL SPATIAL DOMAIN

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In the article we study the problem of continuation of the solution of the ill-posed Cauchy problem for matrix factorizations of the Helmholtz equation in a multidimensional bounded special domain It is assumed that the solution to the problem exists and is continuously differentiable in a closed domain with exactly given Cauchy data. For this case, an explicit formula for the continuation of the solution is established, as well as a regularization formula for the case when, under the indicated conditions, instead of the Cauchy data, their continuous approximations with a given error in the uniform metric are used. An estimate of the stability of the solution to the Cauchy problem in the classical sense will be obtained.

TIME-DEPENDENT INVERSE SOURCE PROBLEM FOR DEGENERATE SUB-DIFFUSION EQUATION

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The main object of the present investigation is the following time-fractional degenerate sub-diffusion equation

$$ {}_{RL}D_{0t}^{\alpha}u(t,x) = t^{\beta}u_{xx}(t,x) + r(t)f(t,x), \tag{1}$$

where α, $\beta \in \mathbb{R}$ such that $0 < \alpha \leq 1$, $\beta \geq 0$ and ${}_{RL}D_{0t}^{\alpha}$ represents the Riemann-Liouville fractional derivative of order α [1], $f(t,x)$ is known, $r(t)$ is unknown functions.

We formulate the following time-dependent inverse source problem for Eq. (1):

Problem: To find a pair of functions $\{u(t,x),\, r(t)\}$ with the following properties:

1) $\{u(t,x),\, r(t)\}$ satisfies Eq. (1) in $\Omega = \{(t,x) : 0 < x < 1,\, 0 < t < T\}$;

2) $u(t,x)$ satisfies initial condition $I_{0t}^{1-\alpha}u(t,x)|_{t=0} = \tau(x)$, $0 \leq x \leq 1$ and boundary conditions $u(t,0) = 0$, $u(t,1) = 0$, $0 \leq t \leq T$ together with overdetermination condition

$$\int\limits_0^1 u(t,x)dx = E(t). \tag{2}$$

Here $\tau(x)$, $E(t)$ are given functions, $I_{0t}^{\alpha}u$ represents the Riemann-Liouville fractional integral of order α [1].

We use the method of separation of variables and then using the solution of the Cauchy problem for degenerate fractional differential equation in time-variable, we represent the solution of the problem via infinite series. In order to find unknown source $r(t)$, we use overdetermination condition (2) and obtain the second kind Volterra integral equation with respect to $r(t)$.

Imposing certain conditions on given data and using the appropriate estimation of the Kilbas-Saigo function [2], we prove a unique solvability of the formulated problem.

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GIBBS MEASURES FOR POTTS MODEL IN BIOLOGY AND PHYSICS

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Gibbs measure (see [1]) is a probability measure which is a generalization of the canonical ensemble to infinite systems. The Gibbs measure gives the probability of the system X being in state x (equivalently, of the random variable X having value x) as

$$\mu(X = x) = \frac{1}{Z(\beta)}\exp(-\beta H(x)),$$

where $H(x)$ is a function from the space of states to the real numbers. The parameter β is (a free parameter) the inverse temperature. The normalizing constant $Z(\beta)$ is the partition function.

In general, the Gibbs measures were proposed by Dobrushin, Lanford, and Ruelle to directly study infinite systems, instead of taking the limit of finite systems:

A measure is a Gibbs measure if the conditional probabilities it induces on each finite subsystem satisfy a consistency condition: if all degrees of freedom outside the finite subsystem are frozen, the canonical ensemble for the subsystem subject to these boundary conditions matches the probabilities in the Gibbs measure conditional on the frozen degrees of freedom.

In the talk the following results will be presented (see [2], [3], [4]):

- The full set of translation-invariant splitting Gibbs measures (TISGMs) for the q-state Potts model on a Cayley tree.
- It is shown that at sufficiently low temperatures number of TISGMs is $2^q - 1$.
- It is proved that there are $[q/2]$ (where $[a]$ is the integer part of a) critical temperatures at which the number of TISGMs changes and give the exact number of TISGMs for each intermediate temperature.
- tree-hierarchy of the set of thermodynamic systems of DNAs.
- Applying translation invariant Gibbs measures of the set of DNAs on the Cayley tree we prove several thermodynamic properties of the DNAs.
- In case of very high and very low temperatures we give stationary distributions and typical (most probable) configurations of the systems.

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ON THE CONTINATION OF THE CAUCHY PROBLEM FOR THE POISSON EQUATION IN BOUNDED DOMAIN

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Poisson equation or potential equation

$$-\triangle U(x) \equiv -\sum_{i=1}^{3} \frac{\partial^2 U}{\partial x_i^2} = f(x), \tag{1}$$

is the classical example for second order elliptic partial differential equations and it is a mathematical model to some important physical phenomena. It has applications in many different areas such as plasma physic, electrocardiography, and corrosion non-destructive evaluation (e.g., [1],[2], [3],[4]).

In this paper, we offer an explicit formula for reconstruction of a solution of the Poisson equation in bounded domain from its values and the values of its normal derivative on part of the boundary, i.e., we give an explicit continuation formula for a solution to the Cauchy problem for the Poisson equation.

Problem. Suppose the Cauchy data for a solution to system (0.1) on the surface S:

$$U(y) = f_1(y), \frac{\partial U(y)}{\partial n} = f_2(y), y \in S \tag{2}$$

where $n = (n_1, n_2, n_3)$ is the unit outward-pointing normal to the surface $\partial\Omega$ at a point y, and f_1, f_2 are continuous vector-functions. Given $f_1(y)$ and $f_2(y)$ on S, find $U(x)$ $x \in \Omega$.

The Cauchy problem (2) for the Poisson equation (1) is well-known to be ill-posed. Hadamard noted that solution to problem is not stable. Possibility of introducing a positive parameter σ, depending on the accuracy of the initial data, was noticed by M. M. Lavrent'ev. Uniqueness of the solution follows from the general theorem by Holmgren.

We suppose that a solution to the problem exists (in this event it is unique) and continuously differentiable in the closed domain and the Cauchy data are given exactly. In this case we establish an explicit continuation formula.

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ON THE SPECTRUM OF THE PAULI PROJECTOR IN A MANY-BODY QUANTUM SYSTEM

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Let $\hat{P}_i$ is a two-body projector on Pauli forbidden state in the i-th two-body subsystem of a many-body quantum system, containing n clusters. Then the number of all two-body projectors is $N = n(n-1)/2$. Let $\hat{P} = \sum_i \hat{P}_i$.

The complete Pauli projector in a many-body quantum system is (see Ref.[1]):

$$\hat{\Gamma} = \sum_{i=1}^{3} \hat{P}_i - \sum_{i \neq j=1}^{3} \hat{P}_i \hat{P}_j + \sum_{i \neq j \neq k=1}^{3} \hat{P}_i \hat{P}_j \hat{P}_k - \cdots . \tag{1}$$

Here two-body projectors do not commute with each other: $\hat{P}_i \hat{P}_j \neq \hat{P}_j \hat{P}_i$ and $\hat{P}_i^2 = \hat{P}_i$. However, they commute with the complete projector: $\hat{P}_i \hat{\Gamma} = \hat{\Gamma} \hat{P}_i = \hat{P}_i$. The sums on the right hand side of Eq.(1) contain terms like $\hat{P}_1 \hat{P}_2 \hat{P}_1$, $\hat{P}_1 \hat{P}_2 \hat{P}_3$, $\hat{P}_2 \hat{P}_1 \hat{P}_3$, $\hat{P}_1 \hat{P}_2 \hat{P}_1 \hat{P}_2$, $\hat{P}_1 \hat{P}_2 \hat{P}_3 \hat{P}_1$ etc. due to above noncommutativity. A way to relate the spectrum of the complete projector $\hat{\Gamma}$ with the sum of the two-body projectors $\hat{P}$ is based on the algebra of the operators $\hat{P}_i$. A final result can be formulated as a

THEOREM 1: The complete many-body projector $\hat{\Gamma}$ is related to the sum of the two-body projectors $\hat{P} = \sum_i \hat{P}_i$ as

$$\hat{\Gamma} = 1 - \lim_{m \to \infty} (1 - \hat{P})^m \tag{2}$$

The relation (2) enables us to define the allowed many-body model space, which corresponds to the kernel of the operator $\hat{\Gamma}$. Thus we come to the

THEOREM 2: The kernel of the operator $\hat{P} = \sum_i \hat{P}_i$ is identical to the kernel of the complete many-body Pauli projector $\hat{\Gamma}$.

The above theorems clarify the role of many-body Pauli forces in a multicluster system. We have developed a method for the three-body quantum system. An extension to the case of quantum systems containing more than 3 particles is straightforward.

The results of present work can be used when solving the Schrödinge equation for many-body quantum systems. Since the kernel of the projecting operator defines an allowed subspace, it is good enough to expand a probe wave function of the many-body Hamiltonian over the eigen states of the operator $\hat{P} = \sum_i \hat{P}_i$ corresponding to its zero eigen-value. The obtained results indicate that a truncation of the allowed model space following the operator $\hat{P}$ is a natural procedure. This is valid even for the case when the operators $\hat{P}_i$ and $\hat{P}_j$ $(i \neq j)$ overlap strongly.

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INTEGRATION OF THE LOADED SINE-GORDON EQUATION WITH A SOURCE

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We are consider the system of equations

$$\begin{cases} u_{xt} = \sin u + \gamma(t) u_x(0,t) u_{xx} + \sum_{k=1}^{2N} \left(f_{k1} g_{k1} - f_{k2} g_{k2}\right), \\ L(t) f_k = \xi_k f_k, \quad L(t) g_k = \xi_k g_k, \quad k = 1,\ 2,\ ...,\ 2N, \end{cases} \tag{1}$$

with the initial condition

$$u(x,0) = u_0(x), \quad x \in R, \tag{2}$$

where $\gamma(t)$- given continuous function, $L(t) = i \begin{pmatrix} \frac{d}{dx} & \frac{u_x}{2} \\ \frac{u_x}{2} & -\frac{d}{dx} \end{pmatrix}$.

In the problem it is considered that $f_k = (f_{k1}(x,t),\ f_{k2}(x,t))^T$ is an eigenvector function of the operator $L(t)$ corresponding to its eigenvalue ξ_k. $g_k = (g_{k1}(x,t),\ g_{k2}(x,t))^T$ is linearly independent with f_k and solution of equation $L(t) g_k = \xi_k g_k$, moreover

$$W\{f_k,\ g_k\} \equiv f_{k1} g_{k2} - f_{k2} g_{k1} = \omega_k(t), \quad k = 1,\ 2,\ ...,\ 2N, \tag{3}$$

where $\omega_k(t)$ - initially given continuous functions of t.

Suppose that the function $u(x,t)$ is sufficiently smooth and tends to its limits rapidly enough when $x \to \pm\infty$ and satisfies the condition

$$u(x,t) \equiv 0 (mod\ 2\pi) \text{ in } |x| \to \infty; \int_{-\infty}^{\infty} \left((1+|x|)\,|u_x(x,t)| + |u_{xx}(x,t)|\right) dx < \infty. \tag{4}$$

In this work, we obtained representations for the solutions $u(x,t)$, $f_k(x,t)$, $g_k(x,t)$, $k = 1,\ 2,\ ...,\ 2N$, of problem (1) - (4) with in the framework of the inverse scattering problem method for the operator $L(t)$.

In work [1] was shown that the sin-Gordon equation with a self-consistent source can be solved using the inverse scattering problem method.

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INTEGRATION OF THE PERIODIC HARRY-DYM EQUATION WITH A SOURCE

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The Harry-Dym equation was firstly introduced by Harry Dym and Martin Kruskal as an evolution equation solvable by a spectral problem based on the string equation [1], and rediscovered in a more general form in the papers [2-3].

In this work we have considered the following N- band potential periodic Harry-Dym equation with a source

$$(q^2(x,t))_t = -2(1/q(x,t))_{xxx} + \frac{1}{q(0,t)} \sum_{n=0}^{2N} \left(2(f_x^2(x,t,\lambda_n))q^2(x,t) + f^2(x,t,\lambda_n)(q_x^2(x,t))\right) \tag{1}$$

with the initial condition

$$q(x,0) = q_0(x) \tag{2}$$

where $q(x,t)$ is periodic in x with period π for all time $q(x,t) = q(x+\pi,t)$ and $q_0(x)$ is given π periodic function. Here,$f(x,t)$is Floquet-Bloch solution for the following equation

$$y''(x,t,\lambda) = -\lambda q^2(x,t)y(x,t,\lambda), \quad \lambda = k^2\,, \tag{3}$$

which is defined by

$$f^{\pm}(x,t,\lambda) = c(x,t,\lambda) + \frac{s_x(\pi,t,\lambda) - c(\pi,t,\lambda) \pm \sqrt{\Delta^2(\lambda)-4}}{2s(\pi,t,\lambda)} s(x,t,\lambda).$$

Here, $c(x,\lambda,t)$ and $s(x,\lambda,t)$ are solutions of the equation (3) with the initial conditions, respectively

$$\begin{cases} c(0,t,\lambda) = 1 \\ c_x(0,t,\lambda) = 0 \end{cases}, \begin{cases} s(0,t,\lambda) = 0 \\ s_x(0,t,\lambda) = 1 \end{cases}$$

and $\Delta(\xi)$is defined by $\Delta(\xi) = c(\pi,t,\lambda) + s_x(\pi,t,\lambda)$.

Our aim is to find the solution $\{q(x,t), f(x,t)\}$ of the considering problem (1)-(3) via the inverse scattering technique.

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THE SOLITON SOLUTIONS FOR THE LOADED NONLINEAR SCHRODINGER EQUATION

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Loaded nonlinear equations have important physical applications. Therefore, it is always interesting to find its soliton solutions. In this work by using Hirota method, the soliton solutions of loaded nonlinear Schrodinger equation are studied.

We consider the following loaded nonlinear Schrodinger equation

$$iu_t + 2\left|u\right|^2 u + u_{xx} + h(t)u_x = 0, \tag{1}$$

where $h(t) = -\gamma(t)u(0,t)$ and $u(x,t)$ is an unknown function in $x \in R$, $t \geq 0$; $\gamma(t)$ - an arbitrary given continuous function.

Bilinear form for the loaded nonlinear Schrodinger equation. We will find the solition solution of the loaded nonlinear Schrodinger equation by use of Hirota method. With the help of the dependent variable transformations

$$u = \frac{g}{f}, \tag{2}$$

the equation (1) can be transformed into the bilinear forms

$$\left\{ \begin{array}{l} iD_t g \cdot f + D_x^2 g \cdot f + h(t) D_x g \cdot f = 0, \\ D_x^2 f \cdot f = 2\bar{g}g, \end{array} \right. \tag{3}$$

where $\bar{g}$ is the complex conjugation of the function g, respectively and Hirota's bilinear operators D_x and D_t are defined by

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n g(x,t) f(x',t')|_{x=x',t=t'}, \tag{4}$$

where the subscripts of the functions f and g define the order of the partial derivatives with respect to x and t.

Equations (3) can be solved by introducing the following power series expansions for f and g:

$$f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)} + \,\dots, g = \chi g^{(1)} + \chi^3 g^{(2)} + \,\dots, \tag{5}$$

where χ is a formal expansion parameter. Substituting functions (5) into equations (3) and equating coefficients of the same powers of χ to zero can yield the recursion relation for $f^{(k)}$ and $g^{(k)}$, $k = 1, 2, \dots$.

We have obtained the one-soliton and two-soliton solutions for the loaded nonlinear Schrodinger equation, by directly applying Hirota's bilinear method. Besides other soliton solutions can also be got by Hirota's bilinear method.

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Section 5: Applied mathematics, information sciences and computational mathematics

THE USAGE OF QUADRATIC CRYPTANALYSIS METHOD FOR DES ENCRYPTION ALGORITHM

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The application of cryptanalysis methods such as linear, differential, linear-differential, algebraic, correlation, slide attack has been studied in testing and evaluating the robustness of modern symmetric encryption algorithms. N. Tokareva [3] presented correlation matrices with large deviations for a 4-bit reflection block using extended quadratic approximation functions based on a modified view of the linear cryptanalysis method algorithm. Issues related to the application of these quadratic approximations in cryptanalysis processes have not been fully resolved.

An algorithm using the linear cryptanalysis method and the nonlinear link approximation equations (quadratic cryptanalysis) is presented by M. Matsui. A modified algorithm for constructing nonlinear correlation matrices and finding a bit of the key with maximum probability for open and ciphertext pairs based on them is presented [2].

In this paper, the application of the quadratic cryptanalysis method to the DES encryption algorithm, including the formation of quadratic correlation equations with high probability on the basis of a $\langle u, v\rangle_k$.

Correlation matrix values for linear and quadratic relationships for S-block representations of the DES standard encryption algorithm, including 12 for S1, 8 for S2, 6 for S3, 10 for S4, 6 for S5, 6 for S6, 7 for S7 and 6 equations for S8 were obtained [1].

As an example, the approximation equations defined for the S4-block reflection were analyzed on the basis of Matsuining's modified algorithm.

1. i = 1, a = 3, π = id; j = 2, b = 5, σ = id; p = 48/64 $k_5 \oplus k_6 = 0$;
2. i = 1,a = 9, π = id; j = 2,b = 5,σ = id; p = 16/64 $k_3 \oplus k_6 = 1$;
3. i = 1, a = 28, π = id; j = 2, b = 5, σ = id; p = 16/64 $k_2 \oplus k_3 \oplus k_4 = 1$;
4. i = 1, a = 43, π = id; j = 2, b = 6, σ = id; p = 16/64 $k_1 \oplus k_3 \oplus k_5 \oplus k_6 = 1$;

If these equations are combined and k_4 and k_6 are accepted as arbitrary variables, then the system has the following four solutions:

(k_1,k_2,k_3,k_4,k_5,k_6) = (0,0,1,0,0,0), (1,1,0,0,1,1), (0,1,1,1,0,0), (1,0,0,1,1,1) .

However, the approximation equations with quadratic input are $i = 3$ and $j = 1$, π substitution id = (1,2,3,4,5,6); P = (1,2,3,6,4,5); P = (1,3,2,4,5,6); P = (1,4,2,5,3,6); P = (1,5,2,4,3,6); P = (1,5,2,6,3,4); In the cases a = 6 and b = 15 there are quadratic correlations with the probability p = 16/64:

1. i = 3, a = 6, π = id; j = 1, b = 13, σ = id; $k_3k_5 \oplus k_3k_6 \oplus k_4k_5 \oplus k_4k_6 \oplus k_3 \oplus k_6 = 1$;
2. i = 3, a = 6, π = (1,2,3,6,4,5); j = 1, b = 13, σ = id; $k_3k_4 \oplus k_3k_5 \oplus k_4k_6 \oplus k_5k_6 \oplus k_3 \oplus k_5 = 0$;
3. i = 3, a = 6, π = (1,5,2,4,3,6); j = 1, b = 13, σ = id; $k_2k_3 \oplus k_2k_6 \oplus k_3k_4 \oplus k_4k_6 \oplus k_2 \oplus k_6 = 1$;
4. i = 3, a = 6, π = (1,5,2,6,3,4); j = 1, b = 13, σ = id; $k_2k_3 \oplus k_2k_4 \oplus k_3k_6 \oplus k_4k_6 \oplus k_2 \oplus k_4 = 1$.

From the above values of the quadratic approximation equations satisfied only the equations with values (k_1,k_2,k_3,k_4,k_5,k_6) = (1,1,0,0,1,1).

The DES standard encryption algorithm combines the approximation equations with probability $p = 16/64$ for block S expressions in three rounds, resulting in equations with a total probability value of $p = 292/512$.

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BRIEF OVERVIEW OF THE GRAMMAR OF THE UZBEK LANGUAGE

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Abstract: The article discusses the significant role of morphemic analysis for modeling the grammatical categories of parts of speech in the Uzbek language in machine translation. Identification of the types of stages of morphological analysis and common paradigms, differences between the source and target languages.

Keywords: natural language, machine translation, analytic forms, automatic morphology.

The grammar has two parts, namely morphology and syntax.

Parts of speech of Uzbek:			
Dominant elements of a sentence		Secondary elements of a sentence	Separated groups of the words
Noun	Adverb	Conjunction	Interjections
Verb	Numeral	Auxiliary (Yuklama)	Imitative words
Adjective	Pronoun	Helping words (ko'makchi)	Modal words

Morphological formal models arise when using phrases and relationships to each other in the text. Formal models always exist in the syntagma. Syntagma is a semantic syntactic unit that expresses some single words as a semantic part of a sentence. The linguistic database includes grammar and vocabulary. Typically, parsing is done in three main phases in an automated process:

1) Parts of speech 2) Parts of the proposal 3) Types of offers

The Uzbek language is a morphologically rich language with nouns, adjectives and verbs varying in case, number and word forms. The Uzbek language has an agglutinative morphology with productive inflectional and derivational suffixes.[1]

In the formal syntax literature, there are two main approaches to case assignment. The first approach, which is mostly related to Noam. Chomsky's work views case as a syntactic phenomenon licensed by the NPS; the second approach, put forward in the work of Alec Marantz, considers case as a post-syntactic, purely morphological phenomenon.[2]

W+A=¿olma+zor,A+W=¿be+foyda,W-W=¿ota-ona,W-u/yu W=¿Erta-yu kech

Due to the lack of grammatical information for natural language processing, it aims to introduce a descriptive language for the linguistic database.

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DIFFERENCE SCHEMES FOR NUMERICAL SIMULATION OF SURFACE ACOUSTIC WAVES

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The following equation is considered in the paper

$$\ddot{u} - c_0^2 \Delta u = m c_0^2 q \Delta \dot{u} - m c_0^2 \Delta \ddot{u} + f(x,t), \tag{1}$$

where u is the density, c_0 is the speed of sound, q is the relaxation time; the first term on the right-hand side is responsible for the attenuation of the sound wave due to thermal conductivity and viscosity, and the second term regulates dispersion effects. Equation (1) describes the propagation of gravitational-gyroscopic waves in dispersive media and it refers to high-order pseudohyperbolic Sobolev-type equations $\ddot{u} = \partial^2 u/\partial t^2$, $\dot{u} = \partial u/\partial t$, $\Delta = \partial^2/\partial x^2$ is the Laplace operator [1]. Approximating equation (1) with initial conditions and some boundary conditions (local or nonlocal ones) in spatial variables by the finite difference method or the finite element method, we obtain the following Cauchy problem:

$$D\ddot{u}_h(t) + B\dot{u}_h(t) + Au_h(t) = f_h, \;\; u_h(0) = u_{0,h}, \;\; \dot{u}_h(0) = u_{1,h}, \tag{2}$$

where $D = E + mc_0^2\Lambda, B = -mc_0^2 q\Lambda, A = -c_0^2\Lambda, \Lambda u_h = -u_{h,x_i\bar{x}_i}$.

Further, two methods of approximation are considered for (2). First, we approximate problem (2) by the difference scheme

$$Dy_{\bar{t}t} + By_{\overset{\circ}{t}} + Ay = \phi, \;\; y^0 = u_{0,h}, \;\; \dot{y}^0 = u_{1,h}\,, y^n \in H_h,$$

where y approximates u_h, $\psi = O\left(\tau^2 + h^2\right)$. The second way is to discretize problem (2) by the finite element method constructed in [2] (all notations are taken from [3]).

$$D_\gamma \dot{y}_t + By_t + Ay^{(0.5)} = \phi_1, \;\; D_\alpha y_t - (\tau^2/12)B\dot{y}_t - \; D_\beta \dot{y}^{(0.5)} = \phi_2, \, y^0 = u_0, \;\; \dot{y}^0 = u_1. \tag{3}$$

Here $D_m = D - m\tau^2 A$, $m = \alpha, \; \beta, \; \gamma$, $\phi_k \approx f, \;\; k = 1,2$. Parameters $\alpha, \; \beta, \; \gamma$ obey the condition of the fourth order approximation $\alpha + \gamma = \beta + 1/6$, and if the condition $\beta - 6\alpha\gamma + 1/40 = 0$ is additionally satisfied, then scheme (3) has the sixth order approximation. Then, stability conditions are obtained and theorems on the convergence of the constructed numerical algorithms are proved. Algorithms for the implementation of the methods were developed and numerical results were obtained, presented in the form of tables and visualization graphs.

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ANALYSIS OF THE CONSTRUCTION OF LOCAL INTERPOLYATION CUBIC SPLINES ON THE BASIS OF DETAILED DATA

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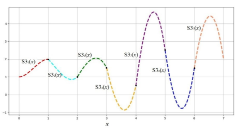
The local interpolation cubic spline function is one of the most pressing issues in the development of science and technology, especially in the application of practical problems.

In particular, in the field of geophysics, biomedicine, environmental processes and other areas, a lot of results are being obtained in the field of signal recovery, processing and forecasting based on spline models.

In this thesis, the construction of a local interpolation cubic spline function based on a linear combination of 2 parabolas with 2 common points was considered, and $S_{3(i)}(x), (i = 1 \dots 7)$ local interpolation cubic spline functions were constructed on the basis of clearly given experimental data and interpolation conditions were checked at connection nodes. The properties of smooth connection at the points of each connection node are studied by plotting graphs of local interpolation cubic spline functions.

$$S_{3(1)}(x_1) = S_{3(2)}(x_1); S_{3(2)}(x_2) = S_{3(3)}(x_2);$$
$$S_{3(3)}(x_3) = S_{3(4)}(x_3); S_{3(4)}(x_4) = S_{3(5)}(x_4);$$
$$S_{3(5)}(x_5) = S_{3(6)}(x_5); S_{3(6)}(x_6) = S_{3(7)}(x_6)$$



The properties of smooth connection at the points of each connection node are studied by plotting graphs of local interpolation cubic spline functions.

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COMPARATIVE ANALYSIS OF METHODS FOR APPROXIMATION OF FUNCTIONS BY POLYNOMIAL SPLINES

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The convenience of calculating the values of splines in computers and their good convergence with their help for widely class types of processes such as interpolation has led to their repeated use.

In contrast to polynomial basis splines, it is possible to represent the initial dependence in the form of sums of paired products of constant coefficients by the values of basis functions. This provides a basis for substantial parallelization of the computation of functions.

The wide popularity of spline approximation methods is explained by the fact that they serve as a universal tool for modeling functions and, in comparison with other mathematical methods, at equal information and hardware costs, provide greater accuracy of calculations. The local interpolation cubic spline of defect 2 considered in this work, which on the segment $[x_i, x_{i+1}]$

has the form:

$$S_3(f,x) = \sum_{j=0}^{3} \varphi_{i+1}(t) f(x_{i+j-1}),$$

here

$$\varphi_1(t) = -\frac{1}{4}t(1-3t+2t^2); \quad \varphi_2(t) = \frac{1}{4}(4-3t-7t^2+6t^3);$$

$$\varphi_3(t) = \frac{1}{4}t(5+5t-6t^2); \quad \varphi_4(t) = -\frac{1}{4}t(1+t-2t^2);$$

$$t = \frac{x-x_i}{h}; \quad h = x_{i+1}-x_i; \quad x \in [x_i, x_{i+1}]; \quad t \in [0,1]$$

.

In what follows, this spline will be denoted by.

$LS_3(x)$ In this work, a comparative analysis (digital processing) by Ryaben'kiy and Grebennikov cubic splines was carried out with specific specified functions. In the course of the study, it was shown that the approximation considered by splines in the work $LS_3(x)$ gives good results. The considered work can be applied to the recovery, processing and prediction of geophysical and biomedical signals.

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CLASSIFICATION OF INFORMATION SECURITY THREATS AND METHODS OF INFORMATION PROTECTION

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Threats of information security can be classified by the following categories: undesirable content; unauthorized access; information leakage; data loss; fraud; cyberwars; cyberterrorism.

In practice, several groups of information protection techniques are used, including:

an obstacle on the path of the intended kidnapper, which is created by physical and software; management, or rendering impact on the elements of the protected system; disguise, or data transformation, usually cryptographic methods; encryption.

To prevent unauthorized access to information, methods such as identification and authentication are used.

$Identification-$ is a mechanism for assigning your own unique name or image to a user who interacts with information.

$Authentication-$ is a system for checking the user's coincidence in the way the tolerance is allowed.

These funds are aimed at providing or, on the contrary, prohibit data admission. The authenticity as the rules is determined by three ways: a program, apparatus, man. In this case, the object of authentication may be not only a person, but also a technical means (computer, monitor, media) or data. The simplest way to protect - password.

Gamming cipher method (additive ciphers) are the most effective in terms of resistance and transformation speed (encryption and decryption procedures). By durability, the data of the ciphers belong to the class of perfect. For encryption and decryption, elementary arithmetic operations are used - open / encrypted message and gamma represented in the numerical form, fold with each other by module (MOD).

$$Ci = (Pi + Ki)modN; Pi = (Ci + N - Ki)modN \tag{1}$$

where Pi, Ci is the $i-$th symbol of the open and encrypted message; $N-$ the number of characters in the alphabet; $Ki - i-$th gamma symbol.

Shifters of the route permutation. The widespread ciphers of rearrangements that use some geometric shape (flat or volumetric) were widespread. The conversion is that the source code fits in the figure along one route, and is written out in a different route.

Cipher tabular route permutation. The largest distribution was the ciphers of the route permutation based on tables. When encrypted in such a table, the source message is enclosed on a specific route, and write down (receive a cipherogram) - a different route. For this cipher, the routes of fitting and writing, as well as the sizes of the table are the key.

Cipher "Crossroads". Figures of a special type can be used for mixing letters. One of these methods is called "Crossroads". The example below draw cross-shaped figures in an amount sufficient to accommodate all the letters of the message. The open text is recorded around these figures in advance specifically - in our case clockwise.

Ciphers using triangles and trapeziums. Help Perform permutations can be both triangles and trapezoids. The open text fits into these figures in accordance with the number of words and the form of the selected figure, which can be stretched or compressed so that the message is placed in it. For the first figure, triangle, the open text is recorded line by the top to the base.

A FREE BOUNDARY PROBLEM FOR A PREDATOR-PREY COMPETITION MODEL

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Population dynamics is one of the most widely discussed topics within biomathematics. The study of the evolution of different populations has always been of special interest, starting with populations of a single species, but evolving to more realistic models where different species live and interact in the same habitat. Among them we can find models that study competitive relationships, symbiosis, commensalism or predator-prey relationships.

The study of the spatial and temporal behavior of a predator and prey in an ecological system is an important issue in population ecology. Various types of mathematical models have been proposed to study the predator-prey system[1-3]. These studies provide a theoretical basis for understanding the complex spatio-temporal dynamics observed in real ecological systems. Such models are mathematically interesting, and rigorous mathematical analysis of these models, such as global existence, uniqueness and stability of solutions, is attracting more and more attention.

In this paper, we consider the following model:

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 u_{xx} + c_1 u_x + u(1-u) - \frac{vu}{u+m} \quad \text{for} \quad t>0 \quad \text{and} \quad 0<x<s(t), \\ \frac{\partial v}{\partial t} = d_2 v_{xx} + c_2 v_x + kv(1-\frac{bv}{u+a}) \quad \text{for} \quad t>0 \quad \text{and} \quad 0<x<s(t), \\ u(0,x)=u_0(x), \quad v(0,x)=v_0(x), \qquad 0\le x\le s_0=s(0), \\ u_x(t,0)=u(t,s(t))=0, \qquad 0\le t\le T, \\ v_x(t,0)=v(t,s(t))=0, \qquad 0\le t\le T, \\ \dot{s}(t) = -\mu\left(u_x(t,s(t)) + \rho v_x(t,s(t))\right), \qquad 0\le t\le T, \end{cases} \tag{1}$$

where $u(t,x)$, $v(t,x)$ denote the population densities of the two competitors, and all parameters are positive numbers.

From a biological point of view, model (1) describes how the two species evolve if they initially occupy the bounded region $[0,h_0]$. The homogeneous Neumann boundary condition at $x=0$ indicates that the left boundary is fixed, with the population confined to move only to right of the boundary point $x=0$. We assume that both species have a tendency to emigrate throught the right boundary point to obtain their new habitat:the free boundary $x=h(t)$ represents the spreading front. Moreover, it is assumed that the expanding speed of the free boundary is proportional to the normalized population gradient at the free boundary. This is well-known as the Stefan condition.

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GENERALIZED MODELS OF THE SEIR EPIDEMIOLOGICAL MODEL AND THEIR DISCRETE ANALOGUES

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Systems of nonlinear differential equations arise in models of population genetics, in particular, in epidemiological issues. These systems describe evolution (the course of a disease over time). However, the discrete version of these equations more adequately describes this process. In the work, we consider the dynamics of discretization of these equations, which are consistent with the Lotka-Volterra model [1].

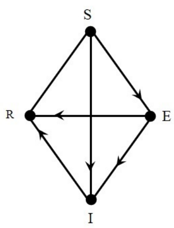
We use in our work a number of concepts from the graph theory [2].

One of the most common models is the SEIR model [3], in which S are susceptible individuals, E are exposed individuals in the stage of incubation (latent) period, I are infected individuals, R are recovered individuals with immunity.

Consider the operator V mapping S^3 to itself with a degenerate skew-symmetric matrix:

$$V:\begin{cases} S^{'}=S(1-aE-bI), \\ E^{'}=E(1+aS-dI-eR), \\ I^{'}=I(1+bS+dE-fR), \\ R^{'}=R(1+eE+fI). \end{cases}$$

Then the corresponding mixed graph has the following form:



Denote by $1\to 2$ transition of individuals from state 1 to state 2. For example, $S\to E$ means the transition of a susceptible individual to a latent one, etc. In the model under consideration, there are the following transitions:

$$S\to E\to R,$$

$$S\to I\to R,$$

$$S\to E\to I\to R.$$

This model corresponds to those cases when the recoveries do not transmit the disease again.

The corresponding theorems and detailed proofs will be given in the forthcoming paper.

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IMPORTANTS OF SECURITY MOBILE HEALTHCARE INFORMATION SYSTEM

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Intellectual property is a legal term that refers to copyright and related rights. It is to play an increasing role in nowadays.

There are several reasons why intellectual property is important to mobile healthcare information system and mobile healthcare information system is important to intellectual property. Mobile healthcare information system, more than other information systems, often involves analysing patients and treatment services that are based on intellectual property and its licensing. Registration, medical history, photos, etc., can all be saved through mobile healthcare information system, in which case, intellectual property is the main component of value in the operations. It is important because the information of value that are saved on the Internet must be protected, using technological security systems and intellectual property laws, or else they can be stolen or pirated and whole information can be destroyed.

Also, intellectual property is involved in making mobile healthcare information system work. The systems that allow the Internet to function—software, networks, designs, chips, routers and switches, the user interface, and so on—are forms of intellectual property and often protected by intellectual property rights.

In this thesis addressed some of the security issues that a manager may face in dealing with information systems that are at the heart of mobile healthcare applications. However mobile application security is an extensive area and under continuous and rapid development. We recommend that managers look at the current trends in technology, and Internet crime. We also recommend that medical organizations have a clear understanding of their risks and the best technologies that can serve as possible countermeasures. One of the approaches to achieve these goals an application security management. This program should include policies, procedures, and audits, as well as technological safeguards such as firewalls, encryption algorithms, authentication devices, intrusion detection systems, and network security management tools.

Managers should continue this evaluation by asking questions such as:

- What could happen and what failures might be expected if the medical organization relies too heavily on information system
- What are the possible risks of losing valuable data and failure of the information system information infrastructure?
- What impact would such a failure have on the work on the whole?
- What are the consequences of such failures in qualitative and quantitative terms?

Finally, should the development of cost models which quantify damages of attacks and the effort of confronting attacks. Such a cost model should be constructed to assess security risk. It helps decision makers to select the appropriate choice of countermeasure(s) to minimize damages/losses due to security incidents.

FRAMEWORK FOR WEBSITE DEVELOPMENT IN PYTHON

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Django is considered the best web framework written in Python. This tool is convenient to use for the development of websites working with databases. Django was created by the developers of the Lawrence-Journal World edition. This newspaper needed a website to publish news on the Internet. Programmers Adrian Golovaty and Simon Willison created a web application and made it public. An active community quickly formed around Django. The framework began to develop rapidly through the efforts of volunteers. A significant role in the success of Django was played by several well-known sites that used this framework. These include Google, Pinterest, Dropbox, Spotify, and The Washington Post website.

Django implements the DRY (don't repeat yourself) principle. Thanks to this, the time for creating websites is reduced. That is, when using Django, you do not need to rewrite the same code several times. The framework allows you to create a website from components. This can be compared to building a fortress with Lego.

Developed ecosystem - Experienced developers recommend to perceive Django as a system. This means that the framework is usually used with a large number of third-party applications. They can be selected depending on the needs of a particular project.

Administrative Panel - The Django Administrative panel is automatically generated when the application is created. This saves the developer from having to create the admin panel manually.

Extensibility - Django functionality is extended with plugins. These are software modules that allow you to quickly add the desired function to the site.

Libraries - Django supports the use of libraries when developing web applications. Popular libraries include:

- Django REST Framework, which simplifies working with the API.
- Django CMS is a convenient tool for content management.
- Django-allauth uses it to implement registration, authorization, and account management functions.

ORM - Django implements object-relational mapping (ORM), which provides application interaction with databases (DB). ORM automatically transfers data from a database, for example, Postgresql or MySQL, to objects that are used in the application code.

Developers choose Django due to such characteristics:

- Separation of business logic and visual part at the architecture level.
- SEO-friendliness
- Extensibility
- Developed infrastructure: a large number of libraries and plugins.
- A large and friendly community, thanks to which it is easy to find answers to difficult questions.

MODELING THE EDUCATIONAL PROCESS

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Creating a single model of different parts of the educational process at the university using different technologies and on this basis to create a convenient system to increase the effectiveness of the teaching process.

Modeling the educational process, many scholars have studied the example of higher education. [1-3].

This work is devoted to create a website for the educational process at the university.We have now developed a base for the higher education process to model the educational process. Database that contains tables such as curriculum, science, direction, form of education, stage of education, semester, science program, staff, structure is suggested. Some of the table relationships in them are proportional and some are disproportionate.

Our website works in a separate mode for professors, teachers, admin and students.We pay special attention to making our website users and create comfortable design.

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MODELING THE PROCESS OF CONTROLLING THE COMPLETENESS OF CERTIFICATION OF GOODS FOR CUSTOMS PURPOSES

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The epidemiological situation prevailing in the world today remains one of the main tasks in the period of the fight against the spread of coronavirus infection, the establishment of social protection of the population and the stability of the functioning of economic sectors. This makes it an urgent issue to ensure strict compliance with the requirements of technical regulations in the conduct of customs clearance of foreign trade goods, complete submission of certificates and permits required for this.

Analysis of the solution to this problem shows that today developed countries more widely implement non-tariff measures to regulate foreign economic activity (FEA), and great importance is given to their simplification. In particular, in the first half of 2021 year 2347 species of technical bar'ers were used in the USA, 227 species in Russia, 182 species in Singapore, 149 species in China.

Today in the customs information system" Single Window " 12 authorized bodies have been established electronic cooperation and 60 types of certificates and permission documents are being formalized electronically. The total number of recorded documents is 471 306.

No matter how convenient and fast the Customs Information System" single window", then the issue of ensuring the completeness of the certification of tokens will remain relevant. Currently, foreign trade operations carried out in the Republic of Uzbekistan are conducted on the basis of the Commodity Nomenclature of Foreign Economic Activity (CN FEA), approved in 2017.

Therefore, in order to solve the issue posed, the authors developed an extended information matrix of CN FEA, in which the code of the goods and the necessary certificate or other permission documents for its customs clearance are fully reflected[1]. Extended information matrix of CN FEA, is implemented on the basis of the information system "single window" of the customs authorities of the Republic of Uzbekistan, which allows remote customs clearance of goods in a global pandemic.

In addition, it allows you to simulate the requirements of national customs legislation on certification of goods, as well as ensuring their completeness.

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APPLICATION OF A DIFFERENCE SPLITTING SCHEME FOR SOLVING PROBLEMS OF A LINEAR TWO-DIMENSIONAL PROBLEM FOR POROUS MEDIA

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We propose a systematic approach to design and investigate the adequacy of the computational models for a mixed dissipative boundary value problem posed for the symmetric t-hyperbolic systems. We consider a two-dimensional linear hyperbolic system with constant coefficients and with dissipative boundary conditions. We construct the difference splitting scheme for the numerical calculation of stable solutions for this system. With the help of the constructed difference scheme, we will carry out computational experiments on the problems of propagation of seismic waves[4].

In the domain $G = \{(t, x, y) : 0 < t \leq T, 0 < x < l, -\infty < y < +\infty\}$, it is considered a hyperbolic system in a special canonical form [1-3],

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{B}\frac{\partial \mathbf{v}}{\partial x} + \mathbf{C}\frac{\partial \mathbf{v}}{\partial y} = f(t, x, y), \tag{1}$$

with boundary conditions for $x = 0$:

$$\mathbf{v}^{\mathbf{I}} = \mathbf{s}\mathbf{v}^{\mathbf{II}}, \tag{2}$$

for $x = l$:

$$\mathbf{v}^{\mathbf{II}} = \mathbf{r}\mathbf{v}^{\mathbf{I}}, \tag{3}$$

and with initial data at $t = 0$

$$v_i(0, x, y) = \varphi_i(x, y), \ \ i = 1, \dots, n, \ \ 0 \leq x \leq l, \ \ -\infty \leq y \leq +\infty \tag{4}$$

where $\mathbf{v}^{\mathbf{I}} = (v_1, v_2, \dots, v_m)^T$, $\mathbf{v}^{\mathbf{II}} = (v_{m+1}, v_{m+2}, \dots, v_n)^T$, $\mathbf{B}$ is a diagonal matrix, $\mathbf{C}$ is a positive definite matrix of order n, $\mathbf{v} = [\mathbf{v}^{\mathbf{I}}, \mathbf{v}^{\mathbf{II}}]^{\mathbf{T}}$, $\mathbf{s}$ is a matrix of order $n-m \times m$, $\mathbf{r}$ is a matrix of order $m \times n-m$. For $|y| > \dfrac{1}{2}Y$ the initial functions are assumed to be zero.

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ASSESSMENT FOR GENERAL CRYPTOGRAPHIC REQUIREMENTS OF S-BLOCK OF THE KUZNECHIK ALGORITHM

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Today, the security of symmetric block encryption algorithms depends on the properties of Boolean functions that implement various cryptographic primitives that are part of them [1].

Information transformation performed by cryptographic primitives can be formalized as a mapping of some space GF(2^n) of n-dimensional vectors over the field GF (2) $X = (x_1, x_2, ..., x_n)$ into another space GF(2^m) m-dimensional binary vectors $Y = (y_1, y_2, ..., y_m)$, where for any $i = 1, 2, ..., n$ and any $j = 1, 2, ..., m, x_i \in$ GF(2), $y_j \in$GF(2).

Mappings of this kind will be specified in the form of a vector Boolean function $Y = \varphi(X)$: GF(2^n)→GF(2^m), which is the union of the component Boolean functions $f_i(X)$ performing the mapping GF(2^n)→GF(2), that is, $\varphi(X) = (f_1(X), f_2(X), ..., f_n(X))$.

To describe Boolean functions, we will use their representation in the form of a truth table and in the form of an algebraic normal form (ANF).

In this work, an assessment is made in accordance with the general cryptographic requirements (Regularity, Non-linearity, Correlation immunity, Algebraic immunity, Disconnected output bits) [2] of S-block of the Kuznechik algorithm [3] of the GOST R 34.12-2015 standard. Based on the results obtained, the followings are proved:

Statement 1. For S-block of the Kuznechik algorithm, the mappings are regular, the common value of nonlinearity is equal to 100, the degrees of algebraic immunity (in all output equations of Boolean functions there are equations of 3 and higher degrees) are equal to 3, the value of output bit unrelatedness (BIC) is 0.

Statement 2. The nonlinearity values $N_(f_1) = 104$, $N_(f_2) = 106$, $N_(f_3) = 116$, $N_(f_4) = 104$, $N_(f_5) = 110$, $N_(f_6) = 106$, $N_(f_7) = 102$, $N_(f_8) = 108$ of the Boolean functions$'$ components S-block of the Kuznechik algorithm for all Boolean functions, the correlation immunity value is equal to 0, and algebraic nonlinearity is equal to 7.

From the above statement, we can conclude that the fixed S-block of the Kuznechik algorithm of the GOST R 34.12-2015 standard are resistant to modern cryptanalysis methods [4].

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TRIANGULAR INEQUALITY – UNWORKABLE SPACE

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This article discusses the space in which the triangle inequality cannot be fulfilled. One of those who posed the problem was Israel Moisovich Gelfand, one of the greatest mathematicians of the twentieth century. These ideas did not escape the attention of eminent mathematicians. In the foreword of the editor of this book, attention is drawn to the problem of Israel Moisovich Gelfand. Indeed, we begin to substantiate the views of Israel Moisovich Gelfand.

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NATURAL LANGUAGE PROCESSING FOR SCARCE TRAINING RESOURCES: THE CASE OF UZBEK

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Recent NLP technologies showing high accuracy results for 10 to 20 high-resource languages with a special focus on English, which is in turn, ignores other low-resource languages[1]. However, it doesn't mean people are not using those languages actively. The Uzbek language is one of the scarce training resources case which is waiting its development. Fortunately, attention for development of the Uzbek are increasing incredibly in the recent five years. In this abstract, I will briefly introduce current efforts so far mafe on Uzbek language as language resource

Current efforts on Uzbek language:

1. The [2] describes **text processing** technologies theoretically. The article deals with the topic of tokenization of text corpora of the Uzbek language, which should take into account the linguistic features of the spelling of the language. **However**, there is still lack well-established practical tokenization library to my knowledge.

2. Several works has been done on **morphological** as well as **syntactic analysis**[3,4,5]. Especially stemming tools has made some practical(pip library, github sources) and theoretical results

3. There is vey good work done recently on **lexical semantics** by creating initial WodNET type thesaurus called UzWordNET(UzW)[6].

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A WEB-APPLICATION INTENDED TO SOLVE A SYSTEM OF LINEAR EQUATIONS

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Millions of people are engaged in mathematical calculations, sometimes because of the attraction to the mysteries of mathematics and its inner beauty, and more often because of professional or other necessity, not to mention study. Many problems lead to the need to solve systems of linear equations. When designing engineering structures, processing measurement results, solving problems of planning the production process and a number of other problems of technology, economics, scientific experiment, it is necessary to solve systems of linear equations. There are many ways to solve systems of equations: addition, substitution, graphical, method of elimination of unknowns, Cramer's method. When solving systems of linear equations at school in algebra lessons, we used methods such as addition, substitution and graphical. Each method is convenient for a specific system.

Cramer's method is used to solve systems of linear algebraic equations in which the number of unknown variables is equal to the number of equations and the determinant of the base matrix which is nonzero.

The geometric interpretation of a system of linear equations in two variables describes a straight line on the coordinate plane Oxy. A system of linear equations in three variables defines a set of planes. The point of intersection is the solution.

Three cases when solving systems of linear equations.

When solving a system of linear equations by Cramer's method, three cases can be encountered:

First case: the system of linear equations has a unique solution (the system is consistent and definite). Conditions: $\Delta \neq 0$. In this case, the straight lines defined by linear equations in two variables intersect at one point.

The second case: the system of linear equations has an infinite set of solutions (the system is consistent and indefinite). Conditions: $\Delta = 0$, $\Delta_{x_1} = \Delta_{x_2} = ... = \Delta_{x_n} = 0$. In this case, the straight lines determined by linear equations in two variables are superimposed on each other.

The third case: the system of linear equations has no solutions (the system is incompatible). Conditions: $\Delta = 0$, $\Delta_{x_1} \neq 0$, $\Delta_{x_2} \neq 0$, ..., $\Delta_{x_n} \neq 0$. In this case, the straight lines defined by linear equations in two variables do not intersect at all.

It is much more difficult to represent these possible cases for a system of linear equations. If these cases are presented visually, that is, in combination with geometric drawings, the study and application of a system of linear equations will be much easier.

Finding a solution to a system of linear equations is a process that requires time-consuming calculations. Therefore, a small admissible arithmetic error in most cases leads to an incorrect solution. In many practical problems, it is not the process of solving a system of linear equations that is important, but how the solution looks. With this in mind, we developed a web application using HTML5, CSS3, Bootstrap and JavaScript technologies to solve a system of two and three unknown linear equations using Cramer's method (Figure 1).

Figure 1. Interface of the web-application.

This program is important in that it provides a solution to a given system of linear equations in a short time, and also shows images that are suitable for solving.

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INTEGRATION OF TRADITIONAL AND COMPUTER GAMES IN THE DEVELOPMENT OF PRESCHOOL CHILDREN

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The term "integration" refers to a community of general scientific concepts. Comprehension of the concept of "integration" came from the comprehension of the categories "connection", "relations", "integrated approach", "system", "integrity". And only in the 80 s of the last century they came to the term "integration".

Pedagogical integration is about establishing connections and relationships between pedagogical means and for the sake of pedagogical goals. [1, p. 19].

From the point of view of the integration of the pedagogical process, the powerful capabilities of pedagogical games. Any game is a deeply integrated form of teaching organization.

The importance of play in the development of a child is great, its researchers argue that "Childhood without play and outside of play is abnormal and immoral" [2, p. 6]. A preschooler who does not know how to play, cannot communicate meaningfully, is not capable of joint activities, is not interested in the problems of his peers. Because of this, he has shortcomings in communicative development (alienation, increased aggressiveness). Thus, play for a child should become "not only a pleasant pastime, but also the dominant technology of education" [3, p. 37].

In today's information and technological era, one of the most modern types of children's games is computer games.

Computer games make it possible to achieve a true revolution in learning: replacing reproductive performance in learning (memorization, acquisition of skills, achievement of automatism in behavior, conditioned reflexes, etc.) with mental development (stimulation of constructive-dialectical and hypothetical-deductive thinking and play activity).

Traditionally, it is believed that the most adequate objects of children's cognitive activity are directly perceived properties and qualities of things, therefore, special attention in preschool childhood is paid to sensory education.

Such computer games developed by us as "Autoworld" [4], "Smart Cards" [5], "Fill in the number" [6], "Basketball" [7], "Funny rings" [8] (a total of 33 such games), have a positive effect on the development of sensory abilities of preschoolers. A beneficial effect on the cognitive development of a preschooler is provided by his knowledge of simple connections and dependencies of the world around him. This tendency in the development of thinking has also been taken into account and reflected in computer games.

Researchers associate the main disadvantages of using a computer as a means of teaching and playing children with the use of imperfect computer programs, the absence of such aspects of interaction as personal communication in the process of a "man-machine" dialogue, and children's excessive enthusiasm for computers. Most of the so-called "commercial" games are also fraught with potential danger: they do not set intellectual tasks for the child and are designed mainly for an external effect, often promoting aggressiveness.

It should be noted that the computer games developed by us are free from the above-mentioned drawbacks. The advantages of their use in the educational process of preschool educational institutions are evidenced by the ongoing research and the presence of psychological and pedagogical expertise. They take into account the age characteristics of children, the needs of the educational process and the existing capabilities of the computer. Certified computer games, used in accordance with existing requirements and recommendations, open up ideal opportunities for the development of methods and organizational forms of education and upbringing of children, enrichment of children's activities and the pedagogical process itself.

Within the framework of this approach, a number of studies have been carried out, in which it is emphasized that computer games do not replace ordinary games, but supplement them, are part of their structure, enriching the pedagogical process with new possibilities. It should be emphasized that for the full use of a computer as a means of play activity, a child needs to be able to operate with symbols (signs), generalized images. In other words, he needs a sufficiently developed thinking, creative imagination, a certain level of arbitrariness of actions. All this is formed in a variety of subject-practical and game activities.

At the same time, the leading activity of a preschooler, a game, is of particular importance for the formation of the need for purposeful computer control, the development of computer games.

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SOME INVARIANT SOLUTIONS OF TWO DIMENSIONAL HEAT EQUATION

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In this paper we give solutions of two-dimensional heat-conductivity equation without a source and drain, which are invariant relative to one one-parameter symmetry group. In this paper it is used the Lie algebra of the infinitesimal generators of the symmetry group of the two-dimensional heat equation, found in [1].

Consider the two-dimensional heat equation

$$u_t = \sum_{i=1}^{2} \frac{\partial}{\partial x_i}(k_i(u)\frac{\partial u}{\partial x_i}) \tag{1}$$

where $u = u(x_1, x_2, t)$-temperature function, $k_i(u) \geq 0$ function of the temperature.

We consider the case $k_1(u) = k_2(u) = u$. In this case, equation (1) has the following form:

$$u_t = u\Delta + (\nabla u)^2 \tag{2}$$

where $\Delta u = \frac{\partial^2 u}{\partial^2 x_1} + \frac{\partial^2 u}{\partial^2 x_2}$ -Laplace operator $\nabla u = \{\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}\}$ gradient of function u.

Symmetry groups and the Lie algebra of infinitesimal generators of the symmetry group for the two-dimensional and three-dimensional heat equation are found in [1]. The Lie algebra of infinitesimal generators of the symmetry group for the one-dimensional heat-conduction equation is found in [2], [3].

One of the Lie infinitesimal generators of the symmetry group for equation (2) is the vector field

$$X = x_1\frac{\partial}{\partial x_1} + x_2\frac{\partial}{\partial x_2} + 2t\frac{\partial}{\partial t}, \tag{3}$$

which generates the following group of symmetries of the space of variables (x_1, x_2, t)

$$(x_1, x_2, t) \to (x_1 e^S, x_2 e^S, te^{2S}),$$

with respect to which the solutions of equation (2) are invariant.It means if $u = u(x_1, x_2, t)$ -solution of equation (2),then for each s function $u = u(x_1 e^{-S}, x_2 e^{-S}, te^{-2S})$ are also solutions of equation (2).

Let us find the invariants of this symmetry group. Function $\xi(x_1, x_2, t)$ is an invariant of the symmetry group if and only if $X(\xi) = 0$. From this condition we find

$$\xi(x_1, x_2, t) = \frac{x_1^2 + x_2^2}{2t}$$

We seek the solution of the equation in the form $u = v(\xi)$. Substituting the derivative functions u into equation (2) we obtain a second-order differential equation with respect to the function $v(\xi)$

$$\xi v'' + \xi v'^2 + vv' + \xi v' = 0 \tag{4}$$

The numerical integration of this equation shows that there is a point ξ_0, which depends on the initial values of the function $v(\xi)$ and its derivative $v'(\xi)$, such that the derivative of the function $v(\xi)$ is equal to zero: $v'(\xi) = 0$. When $\xi \to \xi_0$ function $v(\xi)$ gradually increases and tends to the value $v(\xi_0)$. Thus, when $\xi \to \xi_0$ the temperature is stabilized. Figure 1 shows a graph of the temperature function under the initial conditions $v(0.1) = 0.9$ $v'(0.1) = 10$

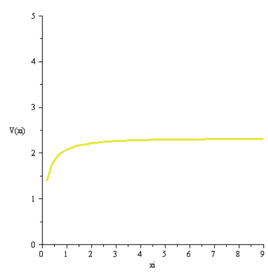

Fig-1

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THE PERIODIC TRANSITION RULE MATRIX ON 2D CA OVER $\mathbb{Z}_2$

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Cellular automata (CA) theory is a very rich and useful model of a discrete dynamical system that focuses on their local information relying on the neighboring cells to produce CA global behaviors. Most of the studies and applications for CA is done for one-dimensional(1D) CA. "The Game of Life" developed by John H.Conway in the 1960s is an example of a two-dimensional (2D) CA. Here, we consider 2D CA with von Neumann neighborhood(see Fig. 1).

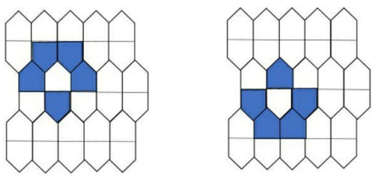

Figure 5.1: Von Neumann neighborhoods for the cells $x_{(3,2)}$ and $x_{(4,2)}$ respectively on the pentagonal lattice model

In this paper, we study 2D CA for von Neumann neighbors on a new pentagonal lattice. This CA is investigated under periodic(CA that boundary cells are adjacent to each other) boundary condition with the 1-state spin value case (i.e. over binary field $\mathbb{Z}_2$).

Here, we give our main result.

Theorem. *Let m be an even positive integer. Then the rule matrix T^P_{Rule} from $\mathbb{Z}_2^{mn}$ to $\mathbb{Z}_2^{mn}$ which takes the tth finite von Neumann CA over pentagonal lattice configuration $C(t)$ of order $m \times n$ to the $(t+1)$th state $C(t+1)$ under periodic boundary condition is obtained by*

$$T^P_{\text{Rule}} = \begin{pmatrix} A & eI & O & O & \dots & O & B \\ lI & A & C & O & \dots & O & O \\ O & B & A & eI & \dots & O & O \\ O & O & lI & A & \dots & O & O \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & O & O & \dots & A & eI \\ C & O & O & O & \dots & lI & A \end{pmatrix}, \tag{1}$$

where all submatrices are $n \times n$, O is the zero matrix, I is the identity matrix and

$$A = \begin{pmatrix} 0&1&0&0&\dots&0&1 \\ 1&0&1&0&\dots&0&0 \\ 0&1&0&1&\dots&0&0 \\ 0&0&1&0&\dots&0&0 \\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots \\ 0&0&0&0&\dots&0&1 \\ 1&0&0&0&\dots&1&0 \end{pmatrix}, \quad B = \begin{pmatrix} 1&1&0&0&\dots&0&0 \\ 0&1&1&0&\dots&0&0 \\ 0&0&1&1&\dots&0&0 \\ 0&0&0&1&\dots&0&0 \\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots \\ 0&0&0&0&\dots&1&1 \\ 1&0&0&0&\dots&0&1 \end{pmatrix}, \ C = \begin{pmatrix} 1&0&0&0&\dots&0&1 \\ 1&1&0&0&\dots&0&0 \\ 0&1&1&0&\dots&0&0 \\ 0&0&1&1&\dots&0&0 \\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots \\ 0&0&0&0&\dots&1&0 \\ 0&0&0&0&\dots&1&1 \end{pmatrix}.$$

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DECORATORS IN THE PYTHON PROGRAMMING LANGUAGE

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Decorators are one of the most important tools in Python and are widely used in the practice of working with functions.

Decorator is a function that allows you to "wrap" to expand its functions without modifying the code of another function.

This allows one program to work with another as if it were self-organized data.

Elements that allow you to perform all the operations that are allowed on objects (passing as a parameter, assigning values ??to variables, retrieving functions, etc.) are called first-class objects.

Functions that take functions as an argument and pass them on to other functions as a result are considered high-order.

In Python, functions are first-class objects, and therefore have the ability to work with higher-order functions.

In higher mathematics, there are a number of higher-order functions similar to the d / dx differential operator. These functions take one function as an argument and return another function as a result. High-order functions in programming work on the same principle.

Since a function is an object of the first category, it can be assigned a value to a variable:

```
def hello_world():
    print('Hello world!')
hello = hello_world
print(helllo())
>>>Hello world
```

This function can also be defined as part of another function. This function encompasses the argument-function and changes its behavior. The decorator returns this shell:

```
def new_func() :
    def hello_world():
        print('Hello world!')
    hello_world()
print(new_func())
>>>Hello world
```

Functions can be passed or received as arguments to other functions:

```
def hello_world():
    print('Hello world!')
def high_degree (func):
    print('As argument from {}'.format(func),' address')
    func()
    return func
high\_degree (hello_world)
```

```
>>>As argument from <function hello_world at 0x0000014FF76B4670>
    address
    Hello world!
```

From the above examples, it can be seen that working with functions in Python is highly organized.

Thus, a decorator is a function that surrounds one function in order to expand its capabilities without modifying the code of another function.

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ELECTRONIC TEXTBOOKS ON PROBABILITY THEORY AND MATHEMATICAL STATISTICS, EDUCATIONAL SYSTEMS (PROGRAMS), MULTIMEDIA APPLICATIONS AND THEIR USE IN THE CLASSROOM

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Resolution of the President of the Republic of Uzbekistan dated September 5, 2018 "On the program of measures to further improve the system of public education of the Republic of Uzbekistan in 2018-2021" No. PP-3931 The task is to improve the quality of education and the introduction of innovative educational technologies. According to him, it is planned to introduce and improve information and communication technologies in the education system, based on best international practices. Also, by the end of 2021, all public education institutions will be provided with broadband Internet (at least 10 Mb / s, taking into account the increase in speed each year by agreement of the parties).

Accordingly, in order to improve the logistics of all educational institutions, new comfortable furniture, modern teaching and laboratory equipment, textbooks and teaching materials, computer and multimedia equipment, video surveillance It is planned to adopt the State Program "Modern School", which provides for the equipment of systems.

Multimedia applications of the electronic textbook prepared for the textbook "Probability Theory and a set of examples and problems from mathematical statistics" - using ICT to cover educational materials in accordance with the state educational standards and curriculum It is aimed at the control and consolidation of knowledge, including animation, tables, texts and dictionaries, which helps students to learn independently, effectively master the subject, including animation, tables, texts and dictionaries, is an interactive electronic information-educational resource that has additional material that enriches the main content of the subject or contains references to similar sources.

Educational-methodical complexes are textbooks, sets of exercises, methodical manuals for teachers, developed in accordance with the state educational standards, curricula and programs, didactic, methodical, pedagogical-psychological, aesthetic and hygienic requirements and including multimedia applications of the textbook. (Intellectual Property Agency of the Republic of Uzbekistan, certificate No.DGU 2021 1282, 23.04.2021.)

These applications are designed to support the teaching process of the new textbook. They can also be used in independent study. Applications can be used to describe new lesson materials, develop students' knowledge and skills, and monitor and evaluate.

The training is conducted independently on the computer. Students can easily solve examples and problems using Mathcad, Excel applications on their chosen topics, as well as use didactic tools, teaching materials or multimedia learning resources.

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CALCULATING Z-NUMBER BY USING THE FUZZY CONVERSION METHOD

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Uncertainty is a common occurrence in life. Most of the information about decision making is uncertain. Humans have a remarkable ability to make wise decisions based on uncertain, unknown, or incomplete information. The creation of this opportunity is a difficult issue [1,2].

This new concept opens up great possibilities for describing human knowledge and is widely used in the process of fuzzy information. Classical fuzzy set theory is relatively advanced and plays an important role in areas such as decision making and fuzzy control [1, 3]. In this case, converting a Z-number to a classical fuzzy number is a rather significant problem.

Method of converting Z-number to classic fuzzy number

The method of converting "Z-number to regular fuzzy number" is performed according to the algorithm described below. Suppose the number Z is given as $Z = (A, B)$. The right side is reliability and the left side is limitations, $A = \{\langle x, \mu_A(x)\rangle \,|x \in [0,1]\}$ and $B = \{\langle x, \mu_B(x)\rangle \,|x \in [0,1]\}$, $\mu_A(x)$ are trapezoidal membership functions, $\mu_B(x)$ is triangle membership function. Here $A = (a_1,\, a_2,\, a_3)$ and $B = (b_1,\, b_2,\, b_3)$ are fuzzy numbers [1,4]. 1) we convert second part (reliability) into a certain number.

$$\alpha = \frac{\int x\mu_B(x)dx}{\mu_B(x)dx},$$

where $\int$ is algebraic integral. 2) The weight of the second part (reliability) is added to the first part (limitation). The weighted number Z can be expressed as follows

$$Z^\alpha = \{\langle x, \mu_{A^\alpha}(x)\rangle \,|\mu_{A^\alpha}(x) = \alpha\mu_A(x),\, x \in [0,1]\},$$

$$E_{A^\alpha}(x) = \alpha E_A(x),\ x \in X, \mu_{A^\alpha}(x) = \alpha\mu_A(x),\ x \in X,$$

$$E_{A^\alpha}(x) = \int_X x\mu_{A^\alpha}(x)dx = \int_X \alpha x\mu_A(x)dx = \alpha\int_X x\mu_A(x)dx = \alpha E_A(x).$$

If the conversion to precision in the fuzzy inference algorithm occurs at the final stage of inference, then in the case of using substituted Z-numbers, the transition to precision occurs ahead of time.

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EXPERIMENTING CIFAR 100/10 BASELINE MODEL AS AN IMAGE CLASSIFICATION PROBLEM

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Advancement of image processing technology has reached very accuracy results and many useful applications has been developed. More specifically, image classification and image recogniton challenges are considered one of the promising and contemporary field. However, there are some communities waiting to use these methods on their own image processing apps. Therefore, we have developed and experimented image classification algorithm with the hope to solve it for local photos.

The following main tasks are considered:
1. Algorithm for image classification using deep neural networks for recognition has been developed.
2. VGG16: Convolutional Network for Classification and Detection deep neural network model for image classification has been developed.
3. CIFAR 10 and CIFAR 100 dataset have been experimented using VGG16 model.

VGG16 experiment shows that the best accuracy for CIFAR 10 is **64%** and for CIFAR 100 is **29%**. Both of the dataset have been trained in 10 epochs. Due to the small training epochs, large-scale dataset doesn't show prominent result. However, it is shown that it allows one to reduce the time spent on processing by reducing the size of the convolutional layers.

The following table shows the result in all layers. The best accuracy result occured on **6, 9, 11** and **14th** layers. The expirement has been carried out in GPU based Nvidia Tesla100 server computer.

Table 5.1: Accuracy table recognition of CIFAR10 and CIFAR100 datasets

Layers	CIFAR10	CIFAR100
21	55%	11%
19	56%	14%
16	58%	17%
14	**60%**	**20%**
11	**62%**	**23%**
9	**64%**	**26%**
6	**57%**	**29%**
3	56%	26%

The result is important for the development of image classification applications for the Uzbek community.

USE MAPLE TO TEACH CURVES OF THE SECOND ORDER

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The use of mathematical software programs of computer technology plays an important role in mathematics. Among the mathematical packages, the Maple program is of special importance due to its ease of use and breadth of functional capabilities. It is a wide-ranging system for working with formulas, numbers, text, and graphics.

The Maple system has a text editor, powerful computing, and a graphics processor. Using its capabilities allows students to visually understand the topic materials and save time spent on calculations.

In particular, the Maple program can be used to teach students the topic of "Curves of the second order" as follows.

Draw a graph of the ellipse with the coordinates of the axes and the center.

> ***with(plottools):***

> ***with(plots):***

> $0 := 3:\ b := 4\ x0 := 0\ y0 := 0$

> ***elli:=ellipse***$([E0, C0], a, b, filled = true, color = blue):$

> ***display(elli,scaling=constrained)***

The rotate command is used to rotate the ellipse graph to a given angle.

>***display***$\left(rotate\left(elli, \frac{\pi}{4}\right)\right)$

In Maple, the $e1 = 25x^2 + y^2 = 25$ algebraic representation of an ellipse is written as follows:

>*with(geometry):*

>*_EnvHorizontalName:='x'; _EnvVerticalName:='y'*

>*ellipse* $\left(e1, 25x^2 + y^2 = 25\right)$*:*

Use the center command to find the coordinates of the center

>*center(e1),coordinates(center(e1))*

$$center\_e1,\ [1, -2]$$

The use of Maple i n the teaching of the second-order curve allows students to get a complete picture of surfaces, to calculate quickly and easily, and to assess the capabilities of information technology.

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CREATION OF MULIMEDIA APPLICATIONS AND THEIR USE IN TEACHING THE DISCIPLINE "DIGITAL AND INFORMATION TECHNOLOGIES"

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The effectiveness and perfection of the learning process depends on how the information is provided to the students and how it is received by the students and how they are used in practice. Multimedia technologies provide future professionals with information about the pedagogical process being studied in animated, audio, and video formats. The most generalized form of the concept of multimedia is the software and hardware for the digital development and creation of text, images, tables, diagrams, photographs, video and audio fragments and various other information. Today, multimedia technologies can be used in various fields of human endeavor, such as business, education, medicine, and so on.

Designing multimedia applications.The content of the multimedia application is analyzed in detail by the author during the preparation of the script and is determined during the development of the technological scenario. The beautifully decorated multimedia application with illustrations, tables and diagrams, animated elements and sound accompaniment facilitates the reception of the studied material, helps to understand and remember, increases the learner's activity in learning, provides a clearer and more complete understanding of the subject.

There are many different technological approaches to developing quality multimedia applications. A number of basic technological recommendations should be followed when creating and using these applications. The basis for the creation of multimedia applications can be considered a model of material coverage, which includes a method of systematization based on the division of material into elements and presentation in a hierarchical manner.

Based on the above requirements, a multimedia electronic complex on the subject of Digital and Information Technologies was created for students of foreign philology. From it, students can learn independent topics using video and animated information, and self-control using thematic tests in the control department.

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USING OF MATHEMATICAL PACKAGES FOR SOLVING APPLIED PROBLEMS

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The expediency of teaching students of technical fields with an innovative pedagogical portfolio method for solving urgent problems of nanoscience in numerical calculations using mathematical packages Maple, Matlab and others is shown.

Today, the discipline "Numerical Methods" is an important discipline in the training of specialists in many areas, and the numerical analysis of mathematical models is an effective research tool in any applied development.

As a working toolkit for solving practical problems in the study of numerical methods, specialized mathematical packages are increasingly used, which allow to intensify the educational process when teaching the course to students of engineering, physics and mathematics, and natural sciences.

Each of the math packages has its own merits. So Maple is very good for symbolic calculations (calculating limits, integrals, derivatives, simplifying algebraic expressions). MatLab - this matrix - oriented language, effective for solving large-scale problems such as solving systems of linear algebraic equations, including the rarefied matrixes, has the tools to work with symbolic mathematics, optimization methods, mathematical statistics, neural networks, etc. The package Mathematica. claims to be a "universal system for mathematics on a computer" and allows you to implement functional programming styles.

Currently, with the development of nanoscience and nanotechnology, it is increasingly necessary to solve problems with many interrelated parameters to which numerical methods are successfully applied using mathematical packages.

We can say that the use of these packages in the research activities of students and researchers in solving applied problems of physics, biology, medicine, materials science, etc. often lead to unexpectedly striking results that are successfully implemented in practice. Usually, in training courses, numerical methods are presented in a concise and accessible level, but not enough attention is paid to solving various modern problems that are relevant from the point of view of applied aspects.

It should be noted that many graduates of higher schools do not know the methods of numerical calculations due to the lack of practical implementation of mathematical packages. This gap can be filled by introducing innovative computer technologies into the educational process, using the method that is most successfully integrated into the student's distance independent work process - the portfolio method.

Our experience at the University gives grounds to assert that a kind of distance work that meets the goals and objectives of competence-based education, namely, promotes the student's initiative in the learning process of self-criticism, readiness to find a way out of various situations, to solve important life and professional problems, is a portfolio for teaching numeric computation methods.

SOFTWARE AND ITS USE FOR THE UZBEK LANGUAGE CORPS

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Abstract

This article describes the system uzbekcorpora.uz for the Uzbek language national corps. The software consists of components such as body search for words and phrases (concordance), markup (word characteristics), lemmas, tokens, and frequency dictionaries. There is also an admin section for editing the corpus database. Uzbekcorpora.uz software is a free online platform. This allows the learner to work anywhere, on any computer, while researching a language.

Recently, software has been developed for teaching languages, automating data in obtaining and researching various statistical analyzes on languages, and working with large amounts of data. In such a situation, the creation of language corps is very important. The 'corpus revolution' took place, as Chapelle described it. Corpus began to be widely used in many fields, including translation, stylistics, grammar, and dictionary creation [2]. John proposed the use of corpses for independent study of how language is used in target contexts in language learning, i.e., an approach to language learning called 'data management' [3].

To work with the Uzbek National Corps, visit http://uzbekcorpora.uz/ using a browser. When you enter the site, the interface shown in Figure 1 is created. The left part of this interface consists of a menu bar that provides general information about the case, project information, and software [3].

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Figure 5.2: Evaluation of materials selection indicators

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OPTIMIZATION OF ELECTRONIC QUEUE RECEIPT FOR DOCTOR'S RECEPTION

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We are deeply concerned about how machine learning and algorithms create and perpetuate inequalities in health. We are to believe that algorithms are developed to ensure that no one will have an unfair advantage over anyone else and that human bias is removed from decision-making. Sounds good in theory.

Scheduling a medical appointment is the most common way for patients to access a health provider: a patient asks for an appointment and is given a day and time to see a doctor. If she's on time, she expects that she'll be seen at or about the time of her appointment.

To maximize efficiency, most outpatient clinics overbook some of their appointment slots, that is, they give the same appointment time to more than one patient. Overbooking is meant to ensure that providers are fully utilized even if some patients fail to show up for their scheduled appointment. However, if patients who are scheduled in overbooked slots do show up, some of them will experience waiting time at the clinic because the provider can see only one patient at a time.

Modern appointment scheduling systems decide which patients to overbook through machine learning: when a patient is given an appointment, a machine-learning algorithm predicts his or her individual probability of showing up for the appointment at the scheduled time—the show-up probability. It can be shown that to maximize efficiency, a clinic should overbook the patients with the lowest show-up probability. the purpose is to optimize provider time and clinic revenue

Significant amounts of data factor into the calculation of a patient's show-up probability: sociodemographic information, the patient's past no-shows, the number of past appointments, how far in advance the appointment is scheduled, and so on.

Our concern is that patients least able to afford waiting are forced to wait longer to be seen by providers and that these patients may in fact leave before being seen, perhaps never to return until their health conditions have worsened. However, their failure to show up for appointments can be conflated with race and incorporated into algorithms in the name of efficiency.

Essentially, instead of optimizing the in-clinic waiting time of the general patient population, our method optimizes the in-clinic waiting time of the group that is worse off. This way, we remove disparities between the different groups.

Our study suggests that there are ways that machine learning and optimization can be used for the benefit of all patients, without leaving anyone behind.

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Section 6: Differential equations, dynamical systems and theory of optimal control

ON A PROBLEM FOR TIME FRACTIONAL ALLER-LYKOV TYPE DIFFERENTIAL EQUATION ON A METRIC STAR GRAPH

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Let $G = (V, B)$ be a graph, consisting of a finite set of vertices (nodes) $V = \{v_i : i = 1, 2, ..., k\}$ and a set of edges B connecting these nodes. The graph considered in this work is a metric graph. Therefore, each edge B_i , $i = 1, 2, ..., k$ is parametrised by an interval $(0, L_i)$.

We consider time fractional Aller-Lykov type equation

$$a \cdot D_{0t}^{\alpha} u^{(i)}(x,t) + D_{0t}^{\alpha-1} u^{(i)}(x,t) = u_{xx}^{(i)}(x,t) - f^{(i)}(x,t) \tag{1}$$

on the each edges ($B_i = \{\, x_i : 0 \le x_i \le L_i \,\}$) of the over defined metric graph , where $1 < \alpha < 2$, $a = const > 0$, $f^{(i)}(x,t)$ are known functions and $D_{ax}^{\alpha} f$ is the Riemann-Liouville fractional derivatives [1].

Problem

To find a solution $u^{(i)}\,(x,t)$ of equation (1) in domain $B_i \times (0, T)$, with the following conditions:

1. solution $u^{(i)}\,(x,t)$ belongs to the class:

$$u^{(i)}\,(x,t) \in C\left([0, L_i] \times (0, T]\right)$$

$$u_{xx}^{(i)}(x,t), {}_{C}D_{0t}^{\alpha} u^{(i)}\,(x,t) \in C\left((0, L_i) \times (0, T)\right),$$

2. takes places initial-boundary and gluing conditions:

$$\lim_{t \to 0} D_{0t}^{\alpha-2} u^{(i)}(x,t) = \tau^{(i)}\,(x)\,,\, x \in \overline{B_i}, \quad \lim_{t \to 0} D_{0t}^{\alpha-1} u^{(i)}(x,t) = \nu^{(i)}\,(x)\,,\, x \in B_i;$$

$$u^{(1)}\,(0,\mathrm{t}) = u^{(2)}\,(0,t) = ... = u^{(k)}\,(0,t)\,, t \in (0, T]\,;$$

$$u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + ... + u_x^{(k)}(0,t) = 0, t \in (0, T)\,;$$

$$u^{(i)}\,(\ell,t) = 0, t \in (0, T]\,, i = 1, 2, 3, ...k,$$

where $\tau^{(i)}\,(x)$ and $\nu^{(i)}\,(x)$ are suffisilently smooth functions.
We use the method of separations of variables for the homogeneous of Eq.(1). By using properties of the Mittag-Lefler function, we prove the uniform convergence of the obtained Fourier series. The uniqueness of the solution of the problem is proved using by A-prior estimation (see [2]).

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ON THE APPROXIMATION OF RENORMALIZATIONS OF INTERVAL EXCHANGE MAPS

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We study generalized interval exchange maps and prove that their Rauzy-Veech renormalizations converge to each other in C^{1+L_1}-norm.

Consider the partition of I into d subintervals indexed by $\mathcal{A}$, that is, $\mathcal{P} = \{I_\alpha,\ \alpha \in \mathcal{A}\}$. Let $f : I \to I$ be a bijection. We say that the triple $(f, \mathcal{A}, \mathcal{P})$ is a **generalized interval exchange map** with d intervals (for short g.i.e.m.), if $f|_{I_\alpha}$ is an orientation-preserving homeomorphism for all $\alpha \in \mathcal{A}$.

Let $f : I \to I$ be a g.i.e.m. with alphabet $\mathcal{A}$ and $\pi_0, \pi_1 : \mathcal{A} \to \{1, ..., d\}$, be bijections such that $\pi_0(\alpha) < \pi_0(\beta)$, iff $I_\alpha < I_\beta$, and $\pi_1(\alpha) < \pi_1(\beta)$, iff $f(I_\alpha) < f(I_\beta)$. We call pair $\pi = (\pi_0, \pi_1)$ the **combinatorial data** associated to the g.i.e.m. f. When appropriate we will also use the notation $\pi = (\pi(1), \pi(2), ..., \pi(d))$ for the combinatorial data of f. We always assume that the pair $\pi = (\pi_0, \pi_1)$ is **irreducible**, that is, for all $j \in \{1, ..., d-1\}$ we have: $\pi_0^{-1}(1, ..., j) \neq \pi_1^{-1}(1, ..., j)$. We say that g.i.e.m. f has **cyclic permutation**, if $\pi_0(\{1, 2, ..., d\}) = \{j+1, ..., d, 1, ..., j\}$, for some $1 \leq j \leq d-1$.

Let us assume that the intervals $I_{\alpha(0)}$ and $f(I_{\alpha(1)})$ have different lengths. Then the g.i.e.m. f is called **Rauzy-Veech renormalizable**(renormalizable, for short). If $|I_{\alpha(0)}| > |f(I_{\alpha(1)})|$ we say that f is renormalizable of **type 0**. When $|I_{\alpha(0)}| < |f(I_{\alpha(1)})|$ we say that f is renormalizable of **type 1**. In either case, the letter corresponding to the largest of these intervals is called **winner** and the one corresponding to the shortest is called the **loser** of π. Let $I^{(1)}$ be the subinterval of I obtained by removing the loser, that is, the shortest of these two intervals:

$$I^{(1)} = \begin{cases} I \setminus f(I_{\alpha(1)}), & \text{if type } 0, \\ I \setminus I_{\alpha(0)}, & \text{if type } 1. \end{cases}$$

The **Rauzy-Veech induction** of f is the first return map $R(f)$ to the subinterval $I^{(1)}$. Denote by $I_\alpha^{(1)}$ the subintervals of $I^{(1)}$. Let f be renormalizable of type 0, then

$$R(f)(x) = \begin{cases} f(x), & \text{if } x \in I_\alpha^{(1)} \text{ and } \alpha \neq \alpha(1), \\ f^2(x), & \text{if } x \in I_{\alpha(1)}^{(1)}. \end{cases} \tag{1}$$

If f is renormalizable of type 1, then

$$R(f)(x) = \begin{cases} f(x), & \text{if } x \in I_\alpha^{(1)} \text{ and } \alpha \neq \alpha(0), \\ f^2(x), & \text{if } x \in I_{\alpha(0)}^{(1)}. \end{cases} \tag{2}$$

It is easy to see, that $R(f)$ is a bijection on $I^{(1)}$ and an orientation-preserving homeomorphisms on each $I_\alpha^{(1)}$. Moreover, the alphabet $\mathcal{A}$ for f and $R(f)$ remains the same.

The triple $(R(f), \mathcal{A}, \mathcal{P}^1)$ is called the **Rauzy-Veech renormalization** of f. We say that g.i.e.m. f **has no connection**, if

$$f^m(\partial I_\alpha) \neq \partial I_\beta, \quad \textit{for all} \quad m \geq 1 \quad \textit{and} \quad \alpha, \beta \in \mathcal{A} \quad \textit{with} \quad \pi_0(\beta) \neq 1. \tag{3}$$

We say that g.i.e.m. $f : I \to I$ has **genus one**(or belongs to the **rotation class**), if f has at most two discontinuities. Note that every g.i.e.m. with either two or three intervals has genus one. The genus of g.i.e.m. is invariant under renormalization.

Denote by B^{KO} the set of g.i.e.m. satisfying the following conditions:

(i) the map f has genus one (cyclic permutation);

(ii) the map f has no connection and has k- bounded combinatorics;

(iii) for each $\alpha \in \mathcal{A}$ we can extend f to $\overline{I}_\alpha$ as an orientation-preserving diffeomorphism satisfying Katznelson and Ornstein's(KO, for short) smoothness condition: f' is absolutely continuous and $f'' \in L_p$, for some $p > 1$.

Our main result is the following

Theorem. Let $f \in \mathbb{B}^{KO}$. Then there exist a sequence of positive numbers $\{\delta_n\} \in l_2$ and an affine i.e.m. $(f_A, \mathcal{A}, \{\widetilde{I}_\alpha\}_{\alpha \in \mathcal{A}})$, that is, $f_A|_{\widetilde{I}_\alpha}$ is affine for each $\alpha \in \mathcal{A}$ such that

(i) f_A has the same combinatorics of f;

(ii) $\|R^n f - R^n f_A\|_{C^1([0,1])} \leq \delta_n, \ \ \|D^2 R^n f - D^2 R^n f_A\|_{L_1([0,1],d\ell)} \leq \delta_n.$

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ON A GENERALIZATION OF THE INITIAL-BOUNDARY VALUE PROBLEM FOR THE EQUATION OF THE BEAM OSCILLATION

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Consider the equation of the beam oscillation in domain $\Omega = \{(x,t) : 0 < x < p, 0 < t < T\}$

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = f(x,t) \tag{1}$$

where $f(x,t)-$ is a given continuous function in $\overline{\Omega}$.

Problem. Find solution $u(x,t) \in C^{4,2}_{x,t}(\overline{\Omega})$, $\frac{\partial^{k+1}u}{\partial t^{k+1}} \in C(\overline{\Omega})$ in domain Ω that satisfies the following conditions

$$u(0,t) = 0, u(p,t) = 0, \ 0 \leq t \leq T \tag{2}$$

$$u_{xx}(0,t) = 0, u_{xx}(p,t) = 0, \ 0 \leq t \leq T \tag{3}$$

$$\frac{\partial^k u}{\partial t^k}(x,0) = \varphi_k(x), \ 0 \leq x \leq p \tag{4}$$

$$\frac{\partial^{k+1} u}{\partial t^{k+1}}(x,0) = \psi_k(x), \ 0 \leq x \leq p \tag{5}$$

where $1 \leq k-$ is a fixed integer.

The uniqueness of the solution to the problem is proved by the spectral method [1]. The existence of the solution to the problem is proved by the Fourier method [2].

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BOUNDARY-VALUE PROBLEM FOR AN INHOMOGENEOUS FOURTH-ORDER EQUATION WITH LOWER TERMS

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For an inhomogeneous fourth-order equation with lower-order terms, a boundary-value problem in a rectangular domain is considered. The uniqueness of the solution is proved by the method of energy integrals. The solution is written out in terms of the constructed Green's function.

For the equation

$$u_{xxxx} + a_1 u_{xx} + a_2 u_x + a_3 u - u_{yy} = f\left(x, y\right), \tag{1}$$

in the region $\Omega = \{(x, y) : 0 < x < p,\ 0 < y < q\}$, we study the following problem.

Problem *A.* Find a function $u\left(x, y\right)$ from class $C^{4,2}_{x,y}(\Omega) \cap C^{3,1}_{x,y}\left(\bar{\Omega}\right)$ satisfying equation (1) in the domain Ω and the following boundary conditions:

$$\alpha u\left(x, 0\right) + \beta u_y\left(x, 0\right) = 0,$$

$$\gamma u\left(x, q\right) + \delta u_y\left(x, q\right) = 0, 0 \le x \le p$$

$$u\left(0, y\right) = \psi_1\left(y\right),\ u\left(p, y\right) = \psi_2\left(y\right),$$

$$u_{xx}\left(0, y\right) = \psi_3\left(y\right), u_{xx}\left(p, y\right) = \psi_4\left(y\right), 0 \le y \le q,$$

where $\psi_i\left(y\right) \in C^3\left[0, q\right], i = \overline{1, 4}, f\left(x, y\right) \in C^{0,1}_{x,y}\left(\bar{\Omega}\right)$, and

$$\alpha\psi_i\left(0\right) + \beta{\psi_i}'\left(0\right) = 0, \quad \gamma\psi_i\left(q\right) + \delta{\psi_i}'\left(q\right) = 0,$$

$${\psi_i}''\left(0\right) = {\psi_i}''\left(q\right) = 0, i = \overline{1, 4}, \quad f\left(x, 0\right) = f\left(x, q\right) = 0.$$

Note that equation (1) was considered in [1], in cases $a_1 = a_2 = a_3 = 0$, and in work [2]: $a_1 = a_2 = 0$, $a_3 = -c\left(x, t\right)$.

Theorem 1. If the problem A has a solution, then under the conditions $a_1 \le 0$, $a_3 \ge 0$, it is unique.

Theorem 2. If inequality

$$C < \frac{\mu_1^3 \left(1 - e^{-2\mu_1 p}\right)^2}{p\left(2\mu_1^2 + 3\mu_1\left(1 + e^{-4\mu_1 p}\right) + 3\right)}$$

holds, then a solution to problem A exists. Here $\mu_1 = \sqrt[4]{\frac{\lambda_1}{4}}$, $C = \max\left\{\left|a_i\right|, i = \overline{1, 3}\right\}$.

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ON SOME BOUNDARY VALUE PROBLEMS FOR EQUATIONS WITH BOUNDARY OPERATORS OF FRACTIONAL ORDER

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We deal with boundary value problems for equations with the operator $\frac{\partial^2}{\partial y^2} - A(x, D)$, where $A(x, D)$ is a nonnegative elliptic differential operator and boundary operators B_y^ρ (see [1]). In particular, boundary conditions can be given through the one-sided Marchaud, Grünwald-Letnikov or Liouville-Weyl fractional derivatives of order ρ. We find orthogonality and smoothness conditions on the boundary function, which guarantee both the existence and uniqueness of the classical solutions.

Consider a boundary value problem

$$u_{yy}(x,y) = A(x,D)u(x,y), \quad x \in \Omega, \quad y > 0, \tag{1}$$

$$B_j u(x,y) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^{\alpha} u(x,y) = 0,\ 0 \le m_j \le m-1,\ j = 1,2,...,l;\ x \in \partial\Omega, \tag{2}$$

$$B_y^{\rho} u(x,+0) = \varphi(x), \quad \rho > 0,\ x \in \Omega, \tag{3}$$

$$|u(x,y)| \longrightarrow 0, \quad y \longrightarrow \infty, \quad x \in \Omega, \tag{4}$$

where $\varphi(x)$ and coefficients $b_{\alpha,j}(x)$ are given functions.

If $\rho = 0$, then we obtain the well-known Dirichlet problem and it is solvable for any continuous boundary functions $\varphi(x)$. If $\rho = 1$, then problem (1) - (4) coincides with the Neumann problem. For the solvability of this problem, it is necessary that the boundary function satisfies the known orthogonality condition. Naturally, the question arises, if ρ varies from zero to one, then from what value of ρ it is necessary to require the orthogonality condition? In S. Umarov [2], it was proved that this exponent is $\frac{1}{2}$. In that work, when the operator A is a Laplace operator, shown problem (1) - (4) is unconditionally solvable for $0 \le \rho < \frac{1}{2}$, and if $\frac{1}{2} \le \rho \le 1$, then the boundary function must satisfy the orthogonality condition.

In this work, for Problem (1) - (4) to have the unique solution, the following orthogonality conditions have been obtained

$$\varphi_k = \int\limits_{\Omega} \varphi(x) v_k(x) dx = 0,\ k = 1,2,...,k_0. \tag{5}$$

where $v_k(x)$ and λ_k the eigenfunctions and eigenvalues of the problem $Av(x) = \lambda v(x)$, $x \in \Omega$; $B_j v(x) = 0$, $x \in \partial\Omega$ and the eigenvalues $\lambda_k = 0$, $k = 1,2,...,k_0$.

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INTEGRATION OF THE SYSTEM OF KAUP EQUATIONS WITH A SELF-CONSISTENT SOURCE

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In [1], it was proved that the equations of Kaup system is completely integrable. This system describes the waves propagation in shallow water. In [2], the complex finite-gap multiphase solutions expressed in terms of the Riemann theta-functions are considered, the multi-soliton solutions are found and the asymptotic behavior of these solutions is studied. In [3-5], the real finite-gap regular solutions of Kaup system were studied. In [6], the 'Inverse Scattering Transform' is used to solve a class of nonlinear equations associated with the inverse problem for the one-dimensional Schrodinger equation with the energy-dependent potential.

In this paper, we consider the Kaup system with a self-consistent source

$$\begin{cases} p_t = -6pp_x - q_x + 2\sum_{n=1}^{N}(\varphi_n^2)_x, \\ q_t = p_{xxx} - 4qp_x - 2pq_x - 2\sum_{n=1}^{N}\left\{-p_x\cdot\varphi_n^2 + (k_n - 2p)(\varphi_n^2)_x\right\}, \\ -\varphi_n'' + q\varphi_n + 2k_n p\varphi_n = k_n^2\varphi_n, n = 1, 2, ..., N \end{cases} \quad (1)$$

under the initial conditions

$$p(x,t)|_{t=0} = p_0(x)\,, \qquad q(x,t)|_{t=0} = q_0(x). \quad (2)$$

Here $p_0(x)$, $q_0(x)$ are real functions and the inequalities hold

$$\int_{-\infty}^{\infty}|p_0(x)|dx < \infty, \quad \int_{-\infty}^{\infty}(1+|x|)[|q_0(x)| + |p_0'(x)|]dx < \infty. \quad (3)$$

The main aim of this study is to find a solution to system (1)-(3) using the method of the inverse scattering problem for the quadratic pencil of Sturm-Liouville equations with decreasing coefficients.

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THE INVERSE PROBLEM FOR DETERMINING THE SOURCE FUNCTION IN THE EQUATION WITH THE RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE

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Let $m-1 < \rho \le m$ and Ω be an arbitrary N-dimensional domain (with a sufficiently smooth boundary $\partial\Omega$). Consider the following problem

$$\begin{cases} \partial_t^\rho u(x,t) - \triangle u(x,t) = f(x),\ x\in\Omega, \quad 0<t<T; \\ Bu(x,t) \equiv \dfrac{\partial u(x,t)}{\partial n} = 0, x\in\partial\Omega, \quad \lim\limits_{t\to 0}\partial_t^{\rho-j}u(x,t) = \varphi_j(x), x\in\overline{\Omega}, j = 1, 2, ..., m \end{cases} \quad (1)$$

where $\varphi_j(x)$ – given functions, Δ – the Laplace operator, ∂_t^ρ – the fractional derivative of the Riemann-Liouville (see. [1]).

In this paper, we investigate the inverse problem of determining the right-hand side of the first equation of the problem (1). For this we need an additional condition

$$u(x,T) = \Psi(x). \quad (2)$$

Theorem. Let $\varphi_j(x)$ and $\Psi(x)$ belong to classes

$$\varphi_j(x) \in W_2^{\left[\frac{N}{2}\right]+1}(\Omega), \quad \Psi(x) \in W_2^{\left[\frac{N}{2}\right]+2}(\Omega), \quad j = 1, 2, ..., m. \quad (3)$$

Moreover, $\varphi_j(x)$, $\triangle\varphi_j(x), \cdots, \triangle^{\left[\frac{N}{4}\right]}\varphi_j(x) \in \dot{W}_2^1(\Omega)$; $\Psi(x)$, $\triangle\Psi(x), \cdots, \triangle^{\left[\frac{N+2}{4}\right]}\Psi(x) \in \dot{W}_2^1(\Omega)$. Then the inverse problem (1)-(2) has a unique solution $\{u(x,t), f(x)\}$ and the form:

$$u(x,t) = \sum_{k=1}^{\infty}\left[\sum_{j=1}^{m}\varphi_{jk}t^{\rho-j}E_{\rho,\rho-j+1}(-\lambda_k t^\rho) + f_k\, t^\rho E_{\rho,\rho+1}(-\lambda_k t^\rho)\right]v_k(x). \tag{4}$$

where are the numbers

$$f_k = \frac{\Psi_k}{T^\rho E_{\rho,\rho+1}(-\lambda_k T^\rho)} - \sum_{j=1}^{m}\frac{\varphi_{jk}E_{\rho,\rho-j+1}(-\lambda_k T^\rho)}{T^j E_{\rho,\rho+1}(-\lambda_k T^\rho)}, \tag{5}$$

and

$$f(x) = \sum_{k=1}^{\infty}\frac{\Psi_k}{T^\rho E_{\rho,\rho+1}(-\lambda_k T^\rho)}v_k(x) - \sum_{k=1}^{\infty}\sum_{j=1}^{m}\frac{\varphi_{jk}E_{\rho,\rho-j+1}(-\lambda_k T^\rho)}{T^j E_{\rho,\rho+1}(-\lambda_k T^\rho)}v_k(x). \tag{6}$$

where $E_{\rho,\mu}$ – the Mittag-Liffler function, φ_{jk} – the Fourier coefficients of the function $\varphi_j(x)$, $v_k(x)$ and λ_k the eigenfunctions and eigenvalues of the problem $-\Delta v(x) = \lambda v(x) \quad x \in \Omega; \quad Bv(x) = 0, \quad x \in \partial\Omega.$

Note, that in the case $0 < \rho \le 1$ it was studied in the work [2].

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UNIQUENESS AND EXISTENCE FOR NON-HOMOGENEOUS EQUATIONS OF THE HILFER TIME-FRACTIONAL DERIVATIVE

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The paper investigates uniqueness and existence of solution for non-homogeneous equations of the Hilfer time-fractional derivative, the elliptic part of which is an elliptic operator of any order, defined in an arbitrary bounded domain Ω with a sufficiently smooth boundary.

Let $A(D) = \sum\limits_{|\alpha|\le m} a_\alpha D^\alpha$ – arbitrary positive formally self-adjoint elliptic differential operator of order $m = 2l$, with constant coefficients a_α, where $\alpha = (\alpha_1, \alpha_2, ..., \alpha_N)$ - multi-index and $D = (D_1, D_2, ..., D_N)$, $D_j = \frac{\partial}{\partial x_j}$.

Let $0 < \beta \le 1$, $0 < \gamma \le 1$ and $0 \le \mu \le 1$. Consider the following mixed problem

$$\begin{cases} D^{(\beta,\gamma)\mu}u(x,t) + A(D)u(x,t) = f(x,t), \quad x \in \Omega, \quad 0 < t \le T; \\ B_j u(x,t) = \sum\limits_{|\alpha|\le m_j} b_{\alpha,j}(x)D^\alpha u(x,t) = 0,\ 0 \le m_j \le m-1,\ j = 1,2,...,l; \quad x \in \partial\Omega; \\ \lim\limits_{t\to+0}\dfrac{d}{dt}I^{(1-\mu)(1-\gamma)}u(x,t) = \varphi(x). \end{cases} \tag{1}$$

where $\varphi(x)$ and coefficients $b_{\alpha,j}(x)$ - given functions, $D^{(\beta,\gamma)\mu}$ – the fractional derivative of Hilfer, I^ρ – the Riemann-Liouville fractional integration of order ρ (see. [1]). As a result, we get a direct initial-boundary value problem.

Theorem. Let conditions $\tau > \frac{N}{2m}$ and $\varphi, f \in D(\hat{A}^\tau)$ be satisfied. Then the mixed problem (1) has a unique solution and it has the form:

$$u(x,t) = \sum_{n=0}^{\infty}\left[\varphi_n t^{(1-\gamma)(\beta-1)}E_{\beta,\beta+\gamma(1-\beta)}(-\lambda_n t^\beta) + \int_0^t f_n(t-\xi)\xi^{\rho-1}E_{\rho,\rho}(-\lambda_n\xi^\rho)d\xi\right]v_n(x),$$

where $\rho=\gamma+\mu(\beta-\gamma)$, $E_{\rho,\mu}$ – the Mittag-Lifffler function, φ_n the Fourier coefficients of the function $\varphi(x)$, $v_n(x)$ and λ_n the eigenfunctions and eigenvalues of the problem $-Av(x)=\lambda v(x) \quad x\in\Omega; \quad B_j v(x)=0, \quad x\in\partial\Omega$.

Note that differential equations with fractional derivatives of Hilfer were studied in the works [1] and [2].

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SOLUTION TO A SYSTEM OF FOURTH-ORDER HYPERGEOMETRIC PARTIAL DIFFERENTIAL EQUATION AND ITS LINEARLY INDEPENDENT SOLUTIONS

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The talk deals with the Kampe de Feriet hypergeometric function in two variables of the fourth order [1]

$$
\begin{aligned}
u\left(x,y\right) &= F^{2;2;2}_{0;3;3}\left[\begin{matrix} a_1,a_2: & b_1,b_2; & c_1,c_2; \\ -: & \beta_1,\beta_2,\beta_3; & \gamma_1,\gamma_2,\gamma_3; \end{matrix}\; x,\, y\right] \\
&= \sum_{m,n=0}^{\infty} \frac{(a_1)_{m+n}(a_2)_{m+n}(b_1)_m(b_2)_m(c_1)_n(c_2)_n}{(\beta_1)_m(\beta_2)_m(\beta_3)_m(\gamma_1)_n(\gamma_2)_n(\gamma_3)_n m!n!} x^m y^n,
\end{aligned}
$$

where $a_1,a_2,b_1,b_2,c_1,c_2,\beta_1,\beta_2,\beta_3,\gamma_1,\gamma_2,\gamma_3\in\mathbb{C}$ and $(a)_k$ ist Pochhammer symbol [2].

Theorem. If $a_1,a_2,b_1,b_2,c_1,c_2,\beta_1,\beta_2,\beta_3,\gamma_1,\gamma_2,\gamma_3\in\mathbb{C}$ and $\beta_1,\beta_2,\beta_3,\gamma_1,\gamma_2,\gamma_3\neq 0$, then function $u(x,y)$ satisfies the following system of hypergeometric partial differential equations of the fourth order

$$
\begin{aligned}
& x^3\left(1-x\right)u_{xxxx} - 2x^3 y u_{xxxy} - x^2y^2u_{xxyy} \\
& + \left\{\beta_1+\beta_2+\beta_3+3-\left[\left(a_1+a_2+1\right)+\left(b_1+b_2+1\right)+4\right]x\right\}x^2u_{xxx} \\
& - \left[a_1+a_2+1+2\left(b_1+b_2+1\right)+4\right]x^2yu_{xxy} - \left(b_1+b_2+1\right)xy^2u_{xyy} \\
& + \left\{\begin{matrix} \left(\beta_1\beta_2+\beta_1\beta_3+\beta_2\beta_3+\beta_1+\beta_2+\beta_3+1\right) \\ -\left[a_1a_2+a_1b_1+a_1b_2+a_2b_1+a_2b_2+b_1b_2+3\left(a_1+a_2+1\right)+3\left(b_1+b_2+1\right)+1\right]x \end{matrix}\right\}xu_{xx} \\
& - \left(a_1b_1+a_1b_2+a_2b_1+a_2b_2+2b_1b_2+a_1+a_2+3b_1+3b_2+3\right)xyu_{xy} - b_1b_2y^2u_{yy} \\
& + \left\{\beta_1\beta_2\beta_3-\left[\left(a_1+1\right)\left(a_2+1\right)\left(b_1+b_2+1\right)+\left(a_1+a_2+1\right)b_1b_2\right]x\right\}u_x \\
& - \left(a_1+a_2+1\right)b_1b_2yu_y - a_1a_2b_1b_2u = 0,
\end{aligned}
$$

$$
\begin{aligned}
& y^3\left(1-y\right)u_{yyyy} - 2xy^3u_{xyyy} - x^2y^2u_{xxyy} \\
& + \left\{\gamma_1+\gamma_2+\gamma_3+3-\left[\left(a_1+a_2+1\right)+\left(c_1+c_2+1\right)+4\right]y\right\}y^2u_{yyy} \\
& - \left(a_1+a_2+1+2\left(c_1+c_2+1\right)+4\right)xy^2u_{xyy} - \left(c_1+c_2+1\right)x^2yu_{xxy} \\
& + \left\{\begin{matrix} \left(\gamma_1\gamma_2+\gamma_1\gamma_3+\gamma_2\gamma_3+\gamma_1+\gamma_2+\gamma_3+1\right) \\ -\left(a_1a_2+a_1c_1+a_1c_2+a_2c_1+a_2c_2+c_1c_2+3\left(a_1+a_2+1\right)+3\left(c_1+c_2+1\right)+1\right)y \end{matrix}\right\}yu_{yy} \\
& - \left(a_1c_1+a_1c_2+a_2c_1+a_2c_2+2c_1c_2+a_1+a_2+3c_1+3c_2+3\right)xyu_{xy} - c_1c_2x^2xu_{xx} \\
& +\gamma_1\gamma_2\gamma_3u_y - \left[\left(a_1+1\right)\left(a_2+1\right)\left(c_1+c_2+1\right)+\left(a_1+a_2+1\right)c_1c_2\right]yu_y \\
& - \left(a_1+a_2+1\right)c_1c_2xu_x - a_1a_2c_1c_2u = 0.
\end{aligned}
$$

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CONSTRUCTION OF A NONTRIVIAL SOLUTION TO THE HOMOGENEOUS CAUCHY PROBLEM FOR ONE EQUATION WITH A FRACTIONAL DERIVATIVE

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Consider in the strip $\Omega = \{(x, y) : -\infty < x < +\infty, 0 < y < T\}$ the Cauchy problem

$$\begin{cases} {}_CD_{0y}^{\frac{1}{2}} u\,(x, y) = (-1)^{n-1} \dfrac{\partial^{2n} u\,(x, y)}{\partial x^{2n}}, \\ \lim\limits_{y \to +0} u\,(x, y) = 0, \end{cases} \tag{1}$$

where

$${}_CD_{0y}^{\frac{1}{2}} u\,(x, y) = \frac{1}{\Gamma\left(\frac{1}{2}\right)} \int\limits_0^y \frac{\frac{\partial u(x,t)}{\partial t} dt}{(y-t)^{\frac{1}{2}}}.$$

In [1], [2] it is shown that an equation of the type (1), for $n = 1$, describes the process of diffusion in fractal media. In [3], the uniqueness of the solution to problem (1) (when $n = 1$, the fractional derivative is of order $0 < \alpha < 1$) was shown in the class of functions satisfying the condition

$$|u\,(x, y)| \leqslant M_1 \exp\left(M_2 |x|^{\frac{2}{2-\alpha}}\right), M_1, M_2 > 0. \tag{2}$$

Also in this paper, an example of non-uniqueness of the solution to the Cauchy problem is given when condition (2) is violated. In this paper, by the method from [4], we construct a nontrivial solution to the problem (1), for which a condition of type (2) is violated.

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THE REMARK ON CONJUGATION OF CIRCLE MAPS WITH BREAK POINTS

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Let f be an orientation preserving homeomorphism of the circle $S^1 \equiv \mathbb{R}/\mathbb{Z}$ with irrational rotation number ρ_f. The classical Denjoy's theorem (see for instance [1]) states that , if f is a circle diffeomorphism with irrational rotation number $\rho = \rho(f)$ and $logDf$ is of bounded variation, then f is conjugate to the linear rotation f_ρ, that is, there exists a homeomorphism f_φ of the circle with $f = \varphi^{-1} \circ f_\rho \circ \varphi$.

It is well known, that for circle homeomorphisms f with irrational rotation number ρ is strictly ergodic i.e. it has a unique f- invariant probability measure μ_f. A remarkable fact is the conjugacy φ can be defined by $\varphi(x) = \mu_f([0, x])$ (see [1]). The last relation shows that the regularity properties of the conjugacy φ imply corresponding properties of the density of the absolutely continuous invariant measure μ_f. This problem of smoothness of the conjugacy of smooth diffeomorphisms is by now very well understood (see for instance [2]). Notice, that for sufficiently smooth circle diffeomorphism with typical irrational rotation number its invariant measure is absolutely continuous with respect to Lebesque measure [2].

A natural extension of circle diffeomorphisms are piecewise smooth homeomorphisms with break points. The regularity properties of invariant measures of of piecewise-smooth circle homeomorphisms are quite different from the case diffeomorphisms (see [3-5]). The invariant measures of piecewise $C^{2+\varepsilon}$ of class piecewise-smooth homeomorphisms with non trivial total jump and with irrational rotation number are singular w.r.t. Lebesgue measure. In this case, the conjugacy φ between f and linear rotation f_ρ is a singular function. Here naturally arises the problem on regularity of conjugacy between two circle maps the same irrational rotation numbers and with break points. This is the rigidity problem for circle homeomorphisms with break points.

In present work we study the conjugations of circle maps with two break points. Denote by $\sigma_1(f_i)$, $\sigma_2(f_i)$ the jump ratio of f_i.

We formulate the main result of our work.

Theorem. Let $f_i \in C^{2+\epsilon}(S^1 \setminus \{a_i, b_i\}), i = 1, 2$ be circle homeomorphisms with two break points with non intersecting orbits and irrational rotation number ρ of "bounded type". Assume

(1) $\sigma_1(f_1) \cdot \sigma_2(f_1) = \sigma_1(f_2) \cdot \sigma_2(f_2) = 1$;

(2) $\mu_1([a_1, b_1]) = \mu_2([a_2, b_2])$, where $\mu_i, i = 1, 2$ are invariant measures of f_i. Then the conjugation ψ between f_1 and f_2 is singular function on the circle i.e. ψ is continues and $\psi'(x) = 0$ a.e. w.r.t Lebesque measure.

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MATHEMATICAL ANALYSIS OF THE CONFLICTING SITUATION OF THE PARTIES WITH HOMOGENEOUS MEANS

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Describe the rate at which a force loses systems as a function of the size of the force and the size of the enemy force. This results in a system of differential equations in force sizes x and y.

$$\begin{cases} \frac{dx}{dt} = -ay \\ \frac{dy}{dt} = -bx \end{cases}$$

The solution to these equations as functions of $x(t)$ and $y(t)$ provide insights about battle outcome. This model underlies many low-resolution and medium-resolution combat models. Similar forms also apply to models of biological populations in ecology.

Lanchester Attrition Model - Square Law. Integrating the equations which describe modern warfare and we get the following state equation, called Lanchester's "Square Law":

$$b(x_0^2 - x^2) = a(y_0^2 - y^2)$$

These equations have also been postulated to describe "aimed fire".

1. $\sqrt{ab}$ measures battle intensity
2. $\sqrt{\frac{a}{b}}$ measures relative effectiveness

Questions Addressed by Square Law State Equation

1. Who will win?
2. What force ratio is required to gain victory?
3. How many survivors will the winner have?
   (a) Basic assumption is that other side is annihilated (not usually true in real world battles)
4. How long will the battle last?
5. How do force levels change over time?
6. How do changes in parameters $x_0,\ y_0,\ a$, and b affect the outcome of battle?
7. Is concentration of forces a good tactic?

After extensive derivation, the following expression for the X force level is derived as a function of time (the Y force level is equivalent):

$$x(t) = \frac{1}{2}\left(x_0 - y_0\sqrt{\frac{a}{b}}\right)e^{\sqrt{ab}\,t} + \frac{1}{2}\left(x_0 + y_0\sqrt{\frac{a}{b}}\right)e^{-\sqrt{ab}\,t}$$

$$y(t) = \frac{1}{2}\left(y_0 - \sqrt{\frac{b}{a}}x_0\right)e^{\sqrt{ab}\,t} + \frac{1}{2}\left(y_0 + \sqrt{\frac{b}{a}}x_0\right)e^{-\sqrt{ab}\,t}$$

Theorem 1. If, according to the quadratic law, one of the following is true. In that case, it is said that the battle was won (lost).

1. To determine who will win, each side must have victory conditions, i.e., we must have a "battle termination model". Assume both sides fight to annihilation.
2. One of three outcomes at time t_f, the end time of the battle:

(a) X wins, i.e., $x(t_f) > 0$ and $y(t_f) = 0$

(b) Y wins, i.e., $y(t_f) > 0$ and $x(t_f) = 0$

(c) Draw, i.e., $x(t_f) = 0$ and $y(t_f) = 0$

3. It can be shown that a Square-Law battle will be won by X if and only if: $\frac{x_0}{x_0} > \sqrt{\frac{a}{b}}$

Force Levels Over Time. At the finishing of the battle, the remaining forces on the X winning side are given by the following formula.

$$x_f = \sqrt{x_0^2 - \frac{a}{b} y_0^2}$$

Here's how it takes X to win.

$$t(x_f) = \frac{1}{2\sqrt{ab}} \ln \left(\frac{1 + \frac{y_0}{x_0}\sqrt{\frac{a}{b}}}{1 - \frac{y_0}{x_0}\sqrt{\frac{a}{b}}} \right)$$

Breakpoint Battle Termination. It will take time for X wins. Assume battle termination at $x(t) = x_{BP}$ or $y(t) = y_{BP}$

$$t(y_{BP}) = \begin{cases} \frac{1}{\sqrt{ab}} \ln \left(\frac{x_0}{x_{BP}} \right) & if \ \frac{x_0}{y_0} = \sqrt{\frac{a}{b}} \\ \frac{1}{\sqrt{ab}} \ln \left(\frac{y_{BP} - \sqrt{y_{BP}^2 - y_0^2 + \frac{b}{a} x_0^2}}{y_0 - x_0 \sqrt{\frac{b}{a}}} \right) & otherwise \end{cases}$$

Theorem 2. If the following condition is met, side X is the winner, otherwise side Y is the winner

$$\frac{x_0}{y_0} > \sqrt{\frac{a}{b} \left(\frac{1 - \frac{y_{BP}^2}{y_0^2}}{1 - \frac{x_{BP}^2}{x_0^2}} \right)}$$

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ON AN ONE NONLINEAR HöLDER TYPE INTEGRAL INEQUALITY AND ITS APPLICATION IN DIFFERENTIAL EVASION GAMES

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Let Ω be the set of continuous nonnegative uniformly restricted functions $u(t)$, such that $0 \leqslant u(t) \leqslant K, t \in [0,1]$, where $K > 0$ is a constant.

Theorem. Let $u(\cdot) \in \Omega$ and satisfy additionally the Hölder type integral inequality

$$u(t) \leqslant f(t) + \int_0^t [a(s) + b(s)u(s) + \sum_{i=1}^{\infty} c_i(s) u^{\alpha_i}(s)] ds \tag{1}$$

where all given functions $f(t), a(t), b(t)$, and $c_i(t)$ are nonnegative, continuous and not equal identically to zero on $[0,1], 0 < \alpha_i < 1, i = 1, 2, ..., n....$

Let us suppose also, that

$g_i(t) = (\int_0^t c_i^{\frac{1}{1-\alpha_i}}(s) ds)^{1-\alpha_i}, g(1) = \gamma_i$ and the series $\sum_{i=1}^{\infty} K^{\alpha_i} \gamma_i$ is convergent. Then there exists a constant $h_0 \in (0;1]$, such that the function $u(t)$ for all $t \in [0; h_0]$ satisfies the inequality

$$u(t) \leqslant f(t) + \int_0^t a(s) ds + \delta \xi_0 + \sum_{i=1}^{\infty} \xi_0^{\alpha_i} g_i(t), \tag{2}$$

where ξ_0 is the positive root of the equation $\xi - \sum_{i=1}^{\infty} d_i \xi^{\alpha_i} - d_0 = 0$
and constants $\delta, h_0, d_i, i = 0, 1, 2, ..,$ are positive and calculated effectively by the functions and the parameters $\alpha_i \in (0;1)$ given in (1).

The inequality (1)-(2) extends [1-2] and successfully used for receiving of lower estimation for the distance of moving position of conflict controlled system to the given terminal set in differential evasion games [3-5]. **References**

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SOLUTION OF THE CAUCHY PROBLEM FOR A SYSTEM OF RIEMANN DIFFERENTIAL EQUATIONS

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The Burgers equation is an example of such model equations which can be represented as a special case of the system [1]

$$\left\{ \begin{array}{l} u_t + uu_x = \nu u_{xx} - b(u - v) \\ v_t + vv_x = \widetilde{\nu} v_{xx} + \tilde{b}(u - v) \end{array} \right. , \tag{1}$$

Consider (1), in the absence of mass forces $(F(t,x) = 0)$ in the band $\Pi_{[0,T]} = \{(t,x) : 0 \le t \le N, x \in 0 < \text{x} \le \text{M}\}$, the initial data of which has the form

$$u|_{t=0} = u_0(x), v|_{t=0} = v_0(x), x \in (0, M]. \tag{2}$$

We will look for solutions of the Cauchy problem of theystem of equations (1) once differentiable in t and twice continuously differentiable in x for $(t,x), v(t,x) \in C^{1,2}\left(\Pi_{[0,T]}\right)$.
System (1) and has a simple solution describing one half-period of a sawtooth wave:

$$\left\{ \begin{array}{ll} u = U(t)\left(1 - \frac{\omega x}{\pi}\right), & 0 < \omega x \le \pi \\ \nu = V(t)\left(1 - \frac{\omega x}{\pi}\right), & 0 < \omega x \le \pi \end{array} \right. , \tag{3}$$

The second half-period is defined as a continuation (3) for the region $-\pi \le \omega x < 0$ in the initial way. Substituting (3) into (1) we obtain a system for amplitudes U(t),V(t)[2].

Thus, the Cauchy problem for a system of Riemann differential equations arising in two-speed hydrodynamics is considered, the system of equations of the Cauchy problem for a system of Riemann differential equations is obtained as a special case of a system of equations of a two-fluid medium.

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ON THE FUNCTIONAL TIGHTNESS OF THE HYPERSPACE OF FINITE DEGREE

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Let X be a topological T_1-space. The set of all non-empty closed subsets of a topological space X is denoted by $\exp X$. The family of all sets of the form

$$O\langle U_1, ..., U_n\rangle = \left\{ F : \ F \in \exp(X),\ F \subset \bigcup_{i=1}^{n} U_i,\ F \bigcap U_i \neq \emptyset,\ i = 1,\ 2,\ ...,\ n \right\}$$

where $U_1, ..., U_n$ are open subsets of X, generates a base of the topology on the set $\exp(X)$. This topology is called the Vietoris topology. The set $\exp(X)$ with the Vietoris topology is called exponential space or the hyperspace of a space X [1]. Put $\exp_n(X) = \{F \in \exp(X) : |F| \leq n\}$ for every natural n.

For the function $f\colon X \to R$, the operation $f_{\exp}\colon \exp_n(X) \to R$ is defined as follows: each set $F \in \exp_n(X)$ is associated with the maximum value functions f on the set F, i.e.

$$f_{\exp}(F) = \max\{f(x) : \ x \in F\}.$$

This operation is defined correctly, since F is a finite set.

Lemma. For any τ-continuous function $f\colon X \to R$, the function

$$f_{\exp}\colon \ \exp_n(X) \to R$$

is τ-continuous.

In [2] A.V.Arhangelskii introduced a cardinal invariant so called the functional tightness of a topological space as follows:

Definition. The *functional tightness* of a space X is

$t_0(X)$=min$\{\tau$: τ is an infinite cardinal and every τ-continuous real-valued function on X is continuous$\}$.

Theorem. For every infinite compact space we have $t_0(X) = t_0(exp_n(X))$.

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ON τ-BOUNDED SPACES

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In this paper we investigate some property of τ-bounded spaces.

Definition 1. [1] Let τ be an infinite cardinal, X and Y topological spaces. A function $\psi\colon X \to Y$ is said to be strictly τ-continuous if for every subspace A of X such that $|A| \leq \tau$, the restriction of ψ to A coincides with the restriction to A of some continuous function $f\colon X \to Y$.

Definition 2. [2] A subset F of a space X is said to be τ-closed if for every $B \subset F$ with $|B| \leq \tau$, the closure in X of the set B is contained in F.

Proposition 1.[2] If a mapping $\psi\colon X \to Y$ is strictly τ-continuous, then for every closed set F in Y, the preimage $\psi^{-1}(F)$ is τ-closed in X.

The following results is a generalization of proposition 1[2]:

Proposition 2. If a mapping $\psi\colon X \to Y$ is strictly τ-continuous, then for every τ-closed set F in Y, the preimage $\psi^{-1}(F)$ is τ-closed in X.

Definition 3.[2] Let τ be an infinite cardinal number. A space X is said to be τ-bounded if the closure in X of every subset of cardinality at most τ is compact.

It is clear that every compact space is τ-bounded for every cardinal number τ.

Proposition 3. Let $f\colon X \to Y$ be a surjective strictly τ-continuous map. If X is τ-bounded, then so is Y.

The main result of this thesis is the following:

Theorem 1. An arbitrary product of τ-bounded spaces is τ bounded **References**

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SOME PROPERTIES OF THE SPACE $C_n(X)$ RELATED TO τ-CLOSED SETS

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In recent times in the investigations [1] and [2] the concept of hyperspace of nonempty closed sets consisting of finitely many of components is introduced. For the space X by $C_n(X)$ denote the set of all closed subsets consisting of no more than n components. This space is good that it contains the hyperspace $\exp_n(X)$ of closed sets consisting of no more than n elements and hyperspace of closed connected sets $\exp{}^c(X)$.

Let X be a topological T_1-space. The set of all non-empty closed subsets of a topological space X is denoted by $\exp X$. The family of all sets of the form

$$O\langle U_1, ..., U_n\rangle = \left\{ F:\ F \in \exp(X),\, F \subset \bigcup_{i=1}^{n} U_i,\, F \bigcap U_i \neq \emptyset,\ i = 1,\, 2,\, ...,\, n \right\}$$

where $U_1, ..., U_n$ are open subsets of X, generates a base of the topology on the set $\exp(X)$. This topology is called the Vietoris topology. The set $\exp(X)$ with the Vietoris topology is called exponential space or the hyperspace of a space X [3]. Put $\exp_n(X) = \{F \in \exp(X) : |F| \leq n\}$, $\exp_\omega(X) = \bigcup\{\exp_n(X) : n = 1, 2, ...\}$, $\exp^c(X) = \{F \in \exp(X) : F \text{ is connected in} X\}$.
It is clear that $\exp_n(X) \subset \exp^c(X) \subset C_n(X) \subset \exp(X)$ for any topological space X. On $C_n(X)$ the topology induced from the hyperspace $\exp(X)$ is considered.

Definition A subset A of a topological space X is called τ-closed [4] if for some $B \subset A$, $|B| \leq \tau$ we have $[B] \subset A$.

Recall that a topological space X is called τ-bounded (see [5]), if the closure in X of every subset of cardinality at most τ is compact.

Theorem 1. Let X be a topological space. Then the set $\Gamma = \{F \in C_n(X)\colon F \cap A \neq \emptyset\}$ is τ-closed, if $A \subset X$ is τ-closed in X.

Theorem 2 Let X be an infinite regular space, then $\cup\beta$ is τ-closed in X for every τ-bounded subspace β of the hyperspace $C_n(X)$.

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ON τ-CLOSED SUBSETS OF HYPERSPACES

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In [1] it was proven that the union of compact subspace in hyperspace is closed in initial topological space. In this thesis our aim is to extend that property for τ-bounded spaces and τ-closed subsets. For more properties of τ-bounded spaces see [2].

The set of all non-empty closed subsets of a topological space X is denoted by $\exp X$. The family of all sets of the form

$$O\langle U_1, U_2, \dots, U_n\rangle = \left\{F : \ F \in \exp X, \, F \subset \bigcup_{i=1}^{n} U_i, \, F \cap U_i \neq \emptyset, \, i = 1, \dots, n\right\}$$

where $U_1, U_2, \dots, U_n$ are open subsets of X, generates a base of the topology on the set $\exp X$. This topology is called the *Vietoris topology.* The set $\exp X$ with the Vietoris topology is called *exponential space* or the *hyperspace of a space X*. [3].

Definition A subset A of a topological space X is called τ-closed [4] if for some $B \subset A$, $|B| \leq \tau$ we have $[B] \subset A$.

Recall that a topological space X is called τ-bounded (see [2]), if the closure in X of every subset of cardinality at most τ is compact.

Theorem 1. Let X be a topological space. Then the set $\Gamma = \{F \in \exp X : F \cap A \neq \emptyset\}$ is τ-closed, if $A \subset X$ is τ-closed in X.

Theorem 2 Let X be an infinite regular space, then $\cup\beta$ is τ-closed in X for every τ-bounded subspace β of the hyperspace $\exp X$.

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SELF-ADJOINT DIRAC OPERATOR ON THE SEMIAXIS IN THE CASE OF FINITE DENSITY

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We consider a system of Dirac differential equations of the form

$$ly = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{dy}{dx} + i\begin{pmatrix} 0 & -\overline{q(x)} \\ q(x) & 0 \end{pmatrix} y = \lambda y, y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, 0 \leq x < \infty$$

with boundary condition

$$y_1(0) = y_2(0),$$

where, the function $q(x)$ satisfies the condition

$$\int_0^{+\infty} (1-x)|q(x)-m|dx < \infty,$$

and $m > 0$ is a mass.

Let us denote by L the self-adjoint Dirac operator generated by the differential expression l in the Hilbert space of two component vector-function

$$L^2((0,\infty), C^2) \equiv L_2^2(0,\infty),$$

with the domain

$$D = \{y = \{y_1, y_2\}^T : y_k \in L^2(0,\infty) \bigcap AC[0,\infty); (ly)_k \in L^2(0,\infty), k = 1,2, y_1(0) = y_2(0)\},$$

where $AC[0,\infty)-$ set of absolutely continuous functions on each finite segment $[0,a] \subset [0,\infty), 0 \le a < \infty$. Wherein, if $y \in D$, then we put $Ly = ly$.

In this work, we study the direct problem of the theory of scattering on the semiaxis for the system of Dirac differential equations in the case of a finite density. The spectrum is investigated, the Riemann surface of the quasimomentum and the resolvent is constructed, and the form of the expansion in eigenfunctions of the self-adjoint Dirac operator is determined.

SOLVABILITY OF A NON-LOCAL PROBLEM WITH AN INTEGRAL CONDITION FOR THE MIXED PARABOLIC–HYPERBOLIC EQUATION

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In the paper we consider equation of mixed parabolic–hyperbolic type

$$\frac{\partial^2 u}{\partial x^2} - \frac{1-signy}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+signy}{2}\frac{\partial u}{\partial y} = 0. \tag{1}$$

in a domain D.

We assum that with $y > 0$ the domain D is bounded by segments AA_1, A_1B_1 and B_1B, where $A = (0,0)$, $A_1 = (0,1)$, $B = (0,1)$, $B_1 = (1,1)$, while with $y < 0$ it is bounded be characteristics

$$AC : x + y = 0, \qquad CB : x - y = 1$$

of equation (1).

The problem. In the domain D find a function $u(x,y)$ such that

$$u(0,y) = \varphi_1(y), \quad u(1,y) = \varphi_2(y), \qquad 0 \le x \le 1, \tag{2}$$

$$\int\limits_0^1 u(x,t)dx = \varphi_3(y) \quad 0 \le t \le T, \tag{3}$$

$$u(x,-x) = \psi(x), \qquad 0 \le x \le 1/2, \tag{4}$$

where $\varphi_i(x)$, $i = \overline{1,3}$ and $\psi(x)$ – given continuous functions.

The existence of a regular solution of the considered problem is proved by using the Green's function and thermal potentials.

The proof is based on the reduction of the problem to the Volterra integral equation of the second kind with a weak feature. It is followed the existence of the unique solution of the considered problem from the solvability of the obtained Volterra integral equations

ON THE INVARIANCE OF A CONSTANT MULTIVALUED MAPPING IN THE HEAT CONDUCTION PROBLEM WITH PULSE CONTROL

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Consider the following heat management problem [1]

$$\frac{\partial u(x,t)}{\partial t} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\Big(a_{ij}(x)\frac{\partial u(x,t)}{\partial x_j}\Big) + F(x,t,\mu),\ 0 < t \le T,\ x \in \Omega \tag{1}$$

with boundary and initial conditions

$$u(x,t) = 0,\ \ 0 \le t \le T,\ x \in \partial\Omega, \tag{2}$$

$$u(x,0) = u^0(x),\ x \in \Omega. \tag{3}$$

Here $u = u(t,x)-$ is an unknown function, T is an arbitrary positive number, $F(x,t,\mu)$ and $u^0(\cdot)-$ are the given functions of their arguments, and $\mu-$ control parameter, where the function F is formulated the general formulation of the problem of control with momentum.

Let $\{t_i\}_{i=0}^{\infty}$, $t_0 > 0-$ be the sequence of moments of time, numbered in ascending order, without finite condensation points. Suppose that the pursuer can act on the system (1) only at the moments $\{t_i\}$ and its influence at these moments has an impulse character, which is expressed by the Dirac delta function [1]:

$$F(x,t,\mu(\cdot)) = \sum_{i=0}^{\infty} \mu(x)\delta(t-t_i),\ x \in \Omega, t \ge 0.$$

Suppose also that the control $\mu(\cdot)$ is a measurable function.

Our further aim is to find such a relationship between the parameters T, b, ρ and λ_i so as to ensure a strong or weak invariance of the map $W(t)$ on the interval $[0,T]$ with respect to problem (1)-(3)[2].

We denote by

$$N(t) = \max\{i \in N \cup \{0\} : t_i \le t \le T\}.$$

Let $\langle u(\cdot,t)\rangle = \|u(\cdot,t)\| = \|u(\cdot,t)\|_{L_2(\Omega)}$, $0 \le t \le T$.

Here $\|u(\cdot,t)\|^2 = \int\limits_{\Omega} |u(\xi,t)|^2\, d\xi = \sum\limits_{k=1}^{\infty} u_k^2(t), 0 \le t \le T$, $u_k(\cdot)-$ the Fourier coefficients of the function $u(\cdot,\cdot)$ in the system $\{\varphi_k(\cdot)\}$.

Theorem 1. 1°. Assume that $t_0 > T$, then for any $\rho \ge 0$ the multivalued mapping $W(t)$ is strongly invariant on the interval $[0,T]$ with respect to problem (1)-(3);

2°. Let's say $t_0 \le T$. If $\rho \le b \cdot \Big(e^{\lambda_1 t_0}\ -1\Big)\Big/\Big(\sum\limits_{i=0}^{N(T)} e^{\lambda_1 t_i}\Big)$, then a multivalued mapping $W(t)$ is strongly invariant on the interval $[0,T]$ with respect to problem (1)-(3).

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A NOTE ON CROSS-RATIO DISTORTION

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The concept of cross-ratio of four consecutively connected segments has its origin in elementary geometry. The question about good estimates on cross-ratio distortion with respect to a smooth function of one variable arose in connection with studies in one-dimensional dynamics. Such tools were developed and applied first to a great success in [1] to the case of critical circle maps and in [2] to the case of unimodal interval maps.There are several cross-ratios used in the study of one-dimensional dynamical systems, all equivalent. In the present paper, we use the following version.

Definition 1. *The **cross-ratio** of four points (z_1, z_2, z_3, z_4), $z_1 < z_2 < z_3 < z_4$ is the number*

$$Cr(z_1, z_2, z_3, z_4) = \frac{(z_2 - z_1)(z_4 - z_3)}{(z_3 - z_1)(z_4 - z_2)}.$$

Definition 2. *Given four real numbers (z_1, z_2, z_3, z_4) with $z_1 < z_2 < z_3 < z_4$ and a strictly increasing function $F : R^1 \to R^1$. The distortion of their cross-ratio under F is given by*

$$Dist(z_1, z_2, z_3, z_4; F) = \frac{Cr(F(z_1), F(z_2), F(z_3), F(z_4))}{Cr(z_1, z_2, z_3, z_4)}.$$

The cross-ratio is preserved by Moebius transformations. More precisely, F is fractional-linear on $[a, b]$ if and only if the cross-ratio distortion of any four distinct points from $[a, b]$ with respect to F is equal to 1.

Now we define cross-ratio distortion for quadruple of points on the unit circle $S^1 \simeq [0, 1)$. For $z_i \in S^1$, $1 \le i \le 4$, suppose that $z_1 \prec z_2 \prec z_3 \prec z_4 \prec z_1$ (in the sense of the ordering on the circle). Then we set $\hat{z}_1 := z_1$ and

$$\hat{z}_i := \begin{cases} z_i, & \text{if } z_1 < z_i < 1, \\ 1 + z_i, & \text{if } 0 < z_i < z_1. \end{cases}$$

for $2 \le i \le 4$. Obviously, $\hat{z}_1 < \hat{z}_2 < \hat{z}_3 < \hat{z}_4$. The vector $(\hat{z}_1, \hat{z}_2, \hat{z}_3, \hat{z}_4)$ is called the lifted vector of $(z_1, z_2, z_3, z_4) \in (S^1)^4$.

Let f be a circle homeomorphism with lift F. We define the cross-ratio distortion of (z_1, z_2, z_3, z_4), $z_1 \prec z_2 \prec z_3 \prec z_4 \prec z_1$ with respect to f by

$$Dist(z_1, z_2, z_3, z_4; f) = Dist(\hat{z}_1, \hat{z}_2, \hat{z}_3, \hat{z}_4; F),$$

where $(\hat{z}_1, \hat{z}_2, \hat{z}_3, \hat{z}_4)$ is the lifted vector of (z_1, z_2, z_3, z_4).

Note that cross-ratio and its distortion plays key role to study the invariant measures and smoothness of conjugation map between two circle homeomorphisms with singularities (i. e. break points or critical points).

Definition 3. *The point $x_{cr} \in S^1$ is called non-flat critical point of a homeomorphism f with order $d \in R$, $d > 2$, if for a some $\omega-$ neighborhood $U_\omega(x_{cr})$, the homeomorphism f can be written as $f(x) = \phi(x)|\phi(x)|^{d-1} + f(x_{cr})$, where ϕ is a C^3 local diffeomorphism with $\phi(x_{cr}) = 0$.*

Now we formulate the main result of the paper.

Theorem 1. *Let f be a circle homeomorphism with critical point x_{cr}. Assume that the interval $[z_1, z_4]$ is a subset of the interval $U_\omega(x_{cr})$ but does not contain a critical point of the homeomorphism f. Let $d = \min\limits_{1 \le s \le 4} \ell([z_s, x_{cr}])$. The following equality holds*

$$Dist(z_1, z_2, z_3, z_4; f) = 1 + O\left(\left(\frac{\ell([z_1, z_4])}{d}\right)^2\right).$$

Note that the similar result to Theorem 1 was obtained by K.M. Khanin [3] for critical homeomorphisms with odd order of critical point.

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ABOUT METHOD FOR SOLVING A SYSTEM OF HYDRAULIC EQUATIONS

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This article discusses a method for solving systems of partial differential equations describing the flow of a viscous incompressible fluid along an inclined plane above a sandy bottom [**?**]. The boundary conditions are periodic in the spatial variable x. After application, the hydrodynamic instability method [**?**], the pseudo-spectral Galerkin method is used for numerical solution. The time derivative is approximated by the Adams – Bashforth difference scheme. The trial solution is represented as a Fourier series. The coefficients of the Fourier series expansion depend on time. The number of decomposition members increases from 32 to 512. One-dimensional flows are considered, i.e., the variables depend only on x and on time t. Considering a particular case when the flow occurs over a solid bottom, a solution was obtained that agrees with the results of [**?**].

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ON THE GEOMETRY OF SUBMERSIONS

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Let $(M, \langle\cdot,\cdot\rangle), (B, \langle\cdot,\cdot\rangle_B)$ be smooth Riemannian manifolds, and let $\pi : (M, \langle\cdot,\cdot\rangle) \to (B, \langle\cdot,\cdot\rangle_B)$ be a smooth mapping of maximal rank. For every $q \in B$ the set $\pi^{-1}(q)$ is a submanifold of M, called fiber of π over q.Vectors from TM tangent to fibers form the smooth vertical distribution; its orthogonal complement with respect to $\langle\cdot,\cdot\rangle)$ is called horizontal distribution. Vectors and vector fields with values in horizontal(resp. vertical) distribution will be called horizontal (resp. vertical).

Definition 1. Let $(M, \langle\cdot,\cdot\rangle), (B, \langle\cdot,\cdot\rangle_B)$ be smooth Riemannian manifolds. A differentiable mapping $\pi : (M, \langle\cdot,\cdot\rangle) \to (B, \langle\cdot,\cdot\rangle_B$ is called a conformal (or: horizontally conformal) submersion if:

1. π is a submersion, i.e. it is surjective and has maximal rank,
2. $\pi_\star$ restricted to horizontal distribution ofπ is a conformal mapping.

With the notation introduced earlier on, the second condition in the above definition can be written in a following way:

$$\exists f \in C^\infty(M) \forall p \in M \forall X, Y \in \mathcal{H}_p \langle X, Y\rangle = e^{2\varphi(p)} \langle \pi_\star X, \pi_\star Y\rangle$$

The function φ will be called the *dilation* of submersion p. A conformal submersion with $\varphi = 0$ is called a *Riemannian submersion* [1].

Let $f : R^n \to R$ be smooth function, and let $|gradf| \neq 0$.

Remark 1. $f : (R^n, g) \to (R, g)$ is a conformal submersion.

Remark 2. $f : (R^n, |\ gradf\ |^2\ g) \to (R, g)$ is a Riemannian submersion.

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Section 7: Theory of probability and mathematical statistics

ON STATISTICAL TOOLS – NEW CONTROL CHARTS THAT ENSURE THE STABILITY OF TECHNOLOGICAL PROCESSES

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It is known that by ensuring the stability of technological processes, the share of defective products in technological operations is reduced and the process can be statistically controlled.

Ensuring the stability of processes using control charts was proposed in 1924 by the American scientist W.E.Shewhart. Then V.E.Deming continued this work, along with the J.Jurans. In the last quarter of the twentieth century, the method of control charts was used in the study of various processes in Asian and European countries, and at the same time this direction is being developed intensively.

Nowadays ISO standards have been developed based on simple Shewhart charts that check the stability of processes. There are various practical methods used in factory practice for technological processes. Conducting research in this area it is proposed a dual control chart "Skewnees-Excess", based on Kolmogorov and Kolmogorov-Smirnov type statistics, which ensures the stability of technological processes, characterised by one or two-dimensional normal distributions [1]. In addition, control charts based on the correlation coefficient are developed that checked the independence or linearity of two-dimensional normal distribution components [2]. In [3, 4] the practical applications of these control charts have been described, which are important for statistically controlling technological processes.

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STOCHASTIC PERTERBUTIONS OF UNSTABLE FEIGENBAUM'S SEPARATRIX

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It is well known(see [1-3]) that Feigenbaum's unstable separatrix $F^{(u)}$ is a family $g(x,t)$ of even, unimodal maps of the segment $[-1,1]$ into itself, analytically depending on x, $|x| < 1+\varepsilon_0$, $\varepsilon_0 > 0$ and t, $|t| < (1-\delta)^{-1}+\varepsilon_0$, $\delta \approx 4,669-$ is the Feigenbaum constant. Let $(\Omega, \mathfrak{F}, P)$ be a probability space and $\{\xi_n\}_{n=1}^{\infty}-$ be a sequence of independent, identically distributed random variables on closed interval $[-\varepsilon, \varepsilon]$, where $\varepsilon > 0$ is parameter. The random variable ξ_n density function $\rho_\varepsilon(x)$. We suppose that $E\xi_n = 0,\ Var\xi_n \sim C\varepsilon^2$, $\varepsilon \to 0,\ C > 0$.

Fix $x_0 \in [-1,1]$. We consider the stationary Markov chain by $x_{n+1} = g(x_n,t) + \xi_{n+1}$ (see [2]) We investigate the stationary distribution of Markov chain as function of ε and t.

Let $n > n_1 - N_0$. We denote

$$\Delta_0^{(k)} = [-\alpha^{-k}, \alpha^{-k}],\ \Delta_i^{(k)}(\tau_n) = g^{(i)}(\Delta_0^{(k)}, \tau_n),\ 1 \le i \le 2^k,\ 1 \le k \le n.$$

The number α is a universal constant of theory of universality, $\alpha \approx 2,5029$ (see [1]-[3]).

There exist m_1, which does not depend on n_1, such that, the intervals $\Delta_i^{(k)}(\tau_n),\ 0 \le i \le 2^k$, are non intersecting , and $\Delta_{2^k}^{(k)} \subset \Delta_0^{(k)}$, if $1 \le k \le n - m_1$. For $n \ge n_1 - m_1$ we define

$$\tilde{\Delta}_i^{(n-l)}(\tau_n) = \{x : dist(x, \Delta_i^{(n-l)}(\tau_n)) \le 4\alpha^{-2l} |\Delta_i^{(n-l)}(\tau_n)|\},\ m_1 \le l < n,$$

$$F(\tau_n, l) = \begin{cases} \bigcup_{i=0}^{2^{n-l}-1} \tilde{\Delta}_i^{(n-l)}(\tau_n) & \text{if } n_1 - m_0 \le n \le n_1, \\ \bigcup_{i=0}^{2^{n_1-l}-1} \tilde{\Delta}_i^{(n_1-l)}(\tau_n) & \text{if } n > n_1. \end{cases}$$

Theorem. *Let $n > n_1 - m_1$ and ν_{ε,τ_n} be the stationary distribution of Markov chain*

$$x_{i+1}(\tau_n) = g(x_i(\tau_n), \tau_n) + \xi_{i+1},\ i = 0,1,2...,$$

where $\tau_n \in (t_n, t_{n+1}]$. Then

$$\nu_{\varepsilon,\tau_n}(F(\tau_n, l)) \ge 1 - \exp\{-R_2(\lambda\alpha^{-2})^l\},\ l \ge m_1$$

where the constant $R_2 > 0-$ does'nt depend on l and n.

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HAUSDORFF DIMENSION OF INVARIANT MEASURES OF PIECEWISE SMOOTH CIRCLE MAPS

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Let μ be a probability measure on a space X. For any set $E \subset X$ and $d > 0$, we define the $d-$dimensional Hausdorff content of E as

$$H_r^d(E) := \inf\left\{\sum_{i=1}^{\infty} r_i^d : E \subset \bigcup_{i=1}^{\infty} B(x_i, r_i), r_i < r\right\}.$$

The $d-$dimensional Hausdorf measure of E is given by $H^d(E) := \sup_{r>0} H_r^d(E)$. The Hausdorf dimension of the set E is defined as $\dim_H E := \inf\{d : H^d(E) = 0\}$. The Hausdorff dimension of the measure μ is defined as

$$\dim_H \mu := \inf\{\dim_H E : \mu(E) = 1\}.$$

In [1], Sadovskaya constructed counterexamples of non-Diophantine rotation numbers. K. Khanin and S. Kocic in [2] proved the following theorem .

Theorem 1.(see [2]) For almost all irrational $\rho \in (0,1)$ and all $\alpha \in (0,1)$ and $c \in R + \backslash\{1\}$, and for any $C^{2+\alpha}-$ smooth circle diffeomorphism with a break f, with a break size of size c and rotation number ρ, we have $\dim_H \mu = 0$, where μ is the unique invariant measure of f.

It is interesting problem is to study the Hausdorff dimension of homeomorphisms with several break points. Denote by $L^{2+\alpha}$, the set of $C^{2+\alpha}-$ smooth circle diffeomorphisms which are C^1 conjugate to piecewise linear homeomorphism with two break points on different orbits.

Next we formulate the main result of our work.

Theorem 2. Let $f \in L^{2+\alpha}$ with rotation number ρ. For almost all irrational numbers $\rho \in (0,1)$, Hausdorff dimension of invariant measure μ of f, $\dim_H \mu = 0$.

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CENTRAL LIMIT THEOREM FOR WEAKLY DEPENDENT RANDOM VARIABLES WITH VALUES IN A HILBERT SPACE.

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Let H be a separable Hilbert space with the norm $\|\cdot\|$. Consider a sequence of $\{X_i, i \geqslant 1\}$ random variables with values in H. The orthonormal basis of the space H is denoted by $\{e_i, i \geq 1\}$ and the decomposition is

$$X_k = \sum_{i=1}^{\infty} X_k^{(i)} e_i$$

We will assume that $\{X_i, i \geqslant 1\}$ satisfies the following dependency condition. For all $k = 1, 2, \cdots$, and $l = 1, 2, \cdots$, and all coordinatewise non-decreasing $f : R^{kl} \to R$ and $g : R^{kl} \to R$ and all disjoints A,B $\subset \{1, 2, \cdots n\}$ the following inequality holds

$$cov(f(X_{t_1}^{(l)}, ..., X_{t_k}^{(l)}), g(X_{m_1}^{(l)}, ..., X_{m_p}^{(l)})) \leq 0$$

where $X_i^{(l)} = (X_{t_1}^{(l)}, ..., X_{t_k}^{(l)}), (t_1, ..., t_k) \in A, (m_1, ..., m_p) \in B.$

A sequence $\{X_i, i \geqslant 1\}$ satisfying this condition is called negatively associated. Let us introduce the notation

$$S_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i,$$

$$t_{i,j} = \lim_{n\to\infty} ES_n^{(i)} S_n^{(j)}.$$

Theorem. Let the stationary sequence $\{X_i, i \geqslant 1\}$ of random variables with values in H satisfy the condition of negative association.Suppose that the following conditions are fulfilled

$$EX_1 = 0, E\|X_1\|^2 < \infty$$

$$\sum_{i=1}^{\infty} t_{ii} < \infty, t_{ii} > 0$$

Then $\{X_i, i \geqslant 1\}$satisfies the central limit theorem i.e. the following weak convergence takes place

$$S_n \Rightarrow N(0, T)$$

where $N(0, T)$ is Gaussian random variable with values in H with zero mean and covariance matrix

$$T = (t_{ij})$$

This result generalizes the central limit theorem proved for one-dimensional negatively associated random variables, see [1].

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MARCINKIEWICZ–ZYGMUND STRONG LAW OF LARGE NUMBERS FOR BANACH SPACE- VALUED MIXING RANDOM VARIABLES

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Let B be a separable Banach space with a norm$\|\cdot\|$.We consider Banach spaces of type p which are defined as following.

Definition 1. A Banach space B(with a norm $\|\cdot\|$) is called type p $(1 \le p \le 2)$ Banach space if for any finite collection of B-valued independent random variables $X_1, X_2, ..., X_n$ with $EX_i = 0$, $E\|X_i\|^p < \infty,\ i = 1, 2, ..., n$ there exists a constant $0 < C(B, p) < \infty$ depending on B and p only such that the following inequality

$$E\left\|\sum_{j=1}^{n} X_j\right\|^p \le C(B,p)\sum_{j=1}^{n} E\|X_j\|^p,$$

holds.

Our main goal is to prove Marcienkiewicz-Zygmund type strong law of large numbers for mixing B-valued random variables (for the case of independent random variables see[1]-[3]).

For the sequence of B-valued random variables $\{X_n,\ n \ge 1\}$ $\psi-$ mixing coefficients are defined as following

$$\psi(k) = \sup\left\{\left|\frac{P(AB) - P(A)P(B)}{P(A)P(B)}\right| : \ A \in F_1^n, B \in F_{n+k}^{\infty},\ n \in N\right\}$$

where $F_a^b - \sigma-$field generated by $X_a, ..., X_b$.

Our main result is the following.

Theorem. Let $\{X_n,\ n \ge 1\}$ be a sequence of identically distributed random variables with values in type p $(1 \le p < 2)$ Banach space B and the following conditions hold:

$$EX_1 = 0,\ \ E\|X_1\|^p < \infty,$$
$$\textstyle\sum_{k=1}^{\infty} \psi(k) < \infty,\ \ \psi(1) < 1.$$

Then as $n \to \infty$,

$$\frac{1}{n^{1/p}}\sum_{i=1}^{n} X_i \to 0, \text{a.s.}.$$

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LAW OF LARGE NUMBERS FOR AUTOREGRESSIVE PROCESSES WITH VALUES IN $D[0,1]$

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Let $\{X_n(t), t \in [0,1], n \geq 1\}$ be a sequence of $D[0,1]$-valued random variables. We will say that the sequence $\{X_n(t), t \in [0,1], n \geq 1\}$, with $EX_k(t) = 0$ satisfies the law of large numbers, if as $n \to \infty$ in $D[0,1]$

$$\tfrac{1}{n}\left(X_1(t) + \ldots + X_n(t)\right) \to 0, \quad \text{in probability},$$

and we say that the sequence satisfies the strong law of large numbers if the above convergence holds almost surely.

Consider a first-order autoregressive process AR (1) defined by the equation

$$Y_{n+1}(t) - \mu(t) = \rho(Y_n(t) - \mu(t)) + X_n(t) \tag{1}$$

where $\mu(t) \in D[0,1]$, $\rho : D[0,1] \to D[0,1]$ linear bounded operator, $\{X_n(t)\}$ sequence of innovation (or innovation process).

Processes of type (1) were studied in detail in monograph [1].First-order autoregressive processes with innovation processes satisfying the mixing conditions were considered in [2].
We denote

$$Z_n(t) = \frac{1}{n}\left(Y_1(t) + ... + Y_n(t)\right).$$

In the talk under certain conditions on the innovation process (see [3]) and operator ρ, we will give the law of large numbers for (1) i.e.,

$$Z_n(t) \to \mu(t), \quad n \to \infty, \text{ in probability}.$$

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THE METHOD OF CORRELATION ANALYSIS IN AGRICULTURE

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In many cases, the experimenter in his studies should be able to determine and evaluate the dependence of the calculated value on one and several random variables.

Two random variables can have either a functional connection, or a correlation connection, or no connection at all. If a change in one quantity entails a change in the average value of another quantity, then they are said to be correlated. For example, the age and weight of a newborn baby, the growth and thickness of young plants, or the yield and the prime cost of crops are correlated.

There exists a rectilinear and curvilinear correlation.

Using the method of correlation analysis, two basic tasks are solved:

(a) the determination of the parameters form of the connection equation,

(b) the measurement of the connection tightness.

The first problem is solved by finding the connection equation and determining its parameters, the second with various indices of connection tightness (correlation coefficient, correlation index, etc.).

The relatively rectilinear and curvilinear correlation connections were studied in [1] for the case when the regression equation has the second-order parabolas. The nonlinear relationship between cow age and productivity was studied in [2].

The harvest yields of winter wheat from seven farms of the area were compared based on the prime cost of 1 centner of grain of this crop.

To study this problem, the method of correlation analysis and the least squares method were used to establish the form, parameters of the equation of connection and the tightness of connection between the random variables under consideration.

Conclusions.

1) The regression equations characterizing the yield and prime cost connection are derived.

2) It is determined that as the yield increases, the prime cost stabilizes around the parameter value of a=4.9.

3) In the case when the results of the experiment show that with the increase in X, the dependent variable Y decreases rapidly, then it is convenient to use the third-order hyperbola equation to flatten the empirical series.

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ABOUT THE USE OF λ - φ DUAL CONTROL CARDS IN THE STATISTICAL ANALYSIS OF PEDAGOGICAL PRACTICES

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It is known that [1] compares the frequencies of the "effect" of interest to the researcher in the two groups observed using Fisher's angle substitution sign (φ^*). In the first case, the sign that indicates the presence or absence of an "effect" may or may not be detected. In [2], we constructed φ control card (φ - CC) that distinguish groups by identifying this character. In the second case, the issue remained open. This work is devoted to the analysis of the second case issue.

Let X and Y numbers in groups characterize the state of the property in which we are interested in. Let $F(x)$ and $G(x)$ be numeric character distributions, and let $F_n(x)$ and $G_n(x)$ be empirical distribution functions corresponding to the selections. In [3], a one-way λ - CC built that sequentially tested the hypotheses H_0: $F(x) > G(x)$ and its alternative H_1: $F(x) \leq G(x)$. In this case, the controlled quantity $\lambda_t = \max\limits_x |F_{n_t} - G_{n_t}(x)|$, ($t = 1, 2, \ldots, k$ – sampling times) is called the "effect" point. If the first group is an experiment and the second is a test group, we use λ_t to divide the group into two groups with "effective- not effective".

We define the percentage of "effective" participants in the experimental group as P_1, and those in the test group as P_2. Using the affirmation in [3], we find the lower control limit of the φ - CC board, which sequentially tests the hypotheses H_0: $P_1 > P_2$ and its alternative H_1: $P_1 \leq P_2$: $LCL_\varphi = 2,31 \cdot \sqrt{\frac{n_1+n_2}{n_1 \cdot n_2}}$, where n_1 is the sample size of X, n_2 is the sample size of Y. In this case, the controlled quantity is $\varphi_t = \varphi_{1t} - \varphi_{2t}$ and is defined as $\varphi_i = 2 \arcsin \sqrt{P_i}$, $i = 1, 2$. During the talk, the status of 6 properties of professional skills on the basis of CEFR is analyzed with a φ - CC based on the results of experiments.

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Section 8: Additional talks

A RELATIONSHIP OF DISCRETE STRUCTURES WITH THE SUBJECT OF COMPUTER SCIENCE

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The science of discrete structures is the theoretical and practical basis of computer science and theoretical informatics. Without studying discrete structures, we cannot understand the content of computer science. The fact that the objects of discrete structures are in the form of a discrete set allows you to clearly express programming problems. Therefore, the importance of discrete structures for undergraduate students majoring in Computer Engineering is important. The basic postulates of discrete structures needed to study in computer science are studied in this article. Many of the core block subjects included in the undergraduate curriculum of computer engineering are inextricably linked to the main concepts of discrete structures and the laws of logic. The science of discrete structures is a course included in the block of mathematical and natural sciences, which forms the theoretical and methodological basis of such disciplines as "Probability Theory and Mathematical Statistics", "Mathematical Modeling ",“Circuits”, “Mathematical and logical bases of digital devices”, “Information security”, “Mobile communication systems”.

The importance and place of teaching a course of discrete structures in the field of computer engineering is determined by three main factors:

1) The constructive part of discrete structures forms the methodological basis of computer science, that is the mathematical core of this subject;

2) Models and methods of discrete structures are used in the study of a large class of economical, biological, chemical, genetic systems, regardless of their physical nature. In particular, discrete models are used to represent data and knowledge in artificial intelligence systems (semantic networks, frame models, declarative knowledge, procedural knowledge, relational graphs, descriptive models, etc.);

3) The language of discrete structures is extremely intelligible and convenient, and it has become a means of describing the concepts of computer science.

Discrete structures allow students to think logically and algorithmically, to master mathematically formalized problems, to solve mathematically formalized problems, to analyze the obtained solutions, to independently study the textbook on discrete structures and their applications , develops the ability to teach the basic concepts of discrete structures in relation to the disciplines of specialization and their application in practice. The combination of discrete structures and computer methods allows the analysis, research and modeling of many economical, technical and technological processes. It follows that the widespread application of the laws of logic in all areas of science and increases its importance.

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LOGICAL DATA MODEL OF INFORMATION SYSTEM FOR THE DEVELOPMENT OF LOGICAL THINKING IN PRESCHOOL EDUCATIONAL INSTITUTIONS

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In modern practice, the performance of any logical task in the teaching of preschool children requires a complex approach, along with entertainment. In this case, it is interesting for children, it attracts them emotionally. The task of the educator is to turn the content of education appropriate to the age characteristics of children into something important for each child. is a strong factor in shaping. The effectiveness of mathematical development in a preschool child determines the success of teaching mathematics in primary school. The study of mathematics in preschool children can not be imagined without the use of didactic games, logical tasks. It is determined taking into account the possibilities.

The child performs logical actions in the development of mental thinking through comparison, analysis, synthesis, classification, generalization.

The article develops an logical data model of information system designed to develop children's logical thinking.

We believe that the only way to solve the problem of accelerating the intellectual development of children in preschool education is the introduction of modern pedagogical and information technologies in the educational process.

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ON AN ALGORITHM FOR DETERMINING PERIODIC OSCILLATING CHEMICAL REACTIONS

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Chemical kinetics is a branch of physical chemistry that studies the mechanism of reactions with understanding the rates of reactions. One of the most important questions of this field is to find the mechanisms of chemical reactions with nonlinear periodic oscillations. So far, such kind of reaction mechanisms have been discovered by researchers, namely, well-known reactions of Belousov-Zhabotinsky, Brusselator, Oregonator.

Note that, having the mechanism of the reaction and using the law of mass actions one can get equation for concentrations of the reactants as dynamical systems. From a mathematical point of view, a reaction with periodic oscillation is equivalent to the presence of a closed trajectory of a dynamical system corresponding to the reaction.

In this work, it is proposed an algorithm to find simple reaction steps for unimolecular, bimolecular and termolecular complex reactions with periodic oscillation. We hope that the reactions we have found will make a scientific contribution to the development of chemical kinetics.

ON AN ALGORITHM FOR DETERMINING CLOSED TRAJECTORIES OF POLYNOMIAL DYNAMICAL SYSTEMS

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It is well-known that mathematical modeling of processes in nature and studying of situations that may occur in the future on the basis of the model are important problems. Since the variables involved in processes are dynamic, i.e. they depend on time, so their mathematical models are often represented by nonlinear dynamic systems, especially polynomial dynamical systems.

Nowadays, the theory of dynamic systems is one of the fastest-growing fields of mathematics. However, despite such a deep development, the quantitative and qualitative study of concrete dynamic systems even on the plane remains one of the most difficult problems. What about the three-dimensional case, the problem is almost unsolved.

One of the important questions of a dynamical system is to answer the existence of a closed trajectory of studying system. From a practical point of view, if the system has a clothed trajectory, then one can conclude about periodic changes of the observed process represented by a given system.

In this work an algorithm for determining clothed trajectories of polynomial dynamical systems of the form

$$\dot{z}_k = \sum_{\substack{\sigma_1+\sigma_2+\sigma_3\leq 3\\ \sigma_1,\sigma_2,\sigma_3\geq 0}} a^{(k)}_{\sigma_1\sigma_2\sigma_3} z_1^{\sigma_1} z_2^{\sigma_2} z_3^{\sigma_3}, \quad a^{(k)}_{\sigma_1\sigma_2\sigma_3} \in \{0, \pm 1\}, \ k = 1, 2, 3$$

is proposed. The algorithm is based on the stability/instability of trajectories of studying dynamical systems.

It should be noted that dynamic systems with closed trajectories that are found by the proposed algorithm can be used to open new types of bifurcations and to find new quantitative and qualitative properties of trajectories in the theory of dynamic systems.

USING COMPUTER SOFTWARE TO LEARN ON MATRICES

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Today, the introduction of computer programs in educational institutions has opened up wide opportunities for optimizing the educational process. In recent years, the use of computers in teaching mathematics has been carried out in several main directions. These include computer assessment of knowledge, that is, computer testing, development and improvement of various training programs, training, development of mathematical games for differentiation, and others.

Experts say that a student who is proficient in mathematics will have a high level of analytical and logical thinking. He develops the ability to quickly make decisions, discuss and negotiate, and act step by step, not only in solving examples and problems, but also in various life situations. Mathematical thinking also brings him to the level of predicting what he wants to do in the future, what is happening around him.

Mathematics plays an important role in developing human intelligence, attention, perseverance and will to achieve a desired goal, maintaining algorithmic discipline, and expanding thinking.

To solve a mathematical problem that arises in many cases quickly and with a given accuracy, a professional mathematician must know a certain algorithmic language and program simultaneously with his profession. For this purpose, mathematical systems more convenient for mathematicians were created in the 90s.

The integration of our country into the world community, the development of science and technology requires the new generation to be competitive in the changing world labor market, mastery of science.

This will be achieved through the introduction of standards in the education system, including the teaching of mathematics, based on the best national and international practices.

We also recommend our program to students to study the topic "Matrices and operations on them" in the course "Higher Mathematics". In this program, it is possible to calculate the sum, difference, multiplication of matrices of size MxN. It takes longer to add, subtract and multiply matrix by matrix. We get faster results by spending less time doing these things with the program!

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CONSTRUCTION OF LOCAL INTERPOLYATION CUBIC SPLINES ON THE BASIS OF ACCURATE DATA PROVIDED IN THE TABLE VIEW

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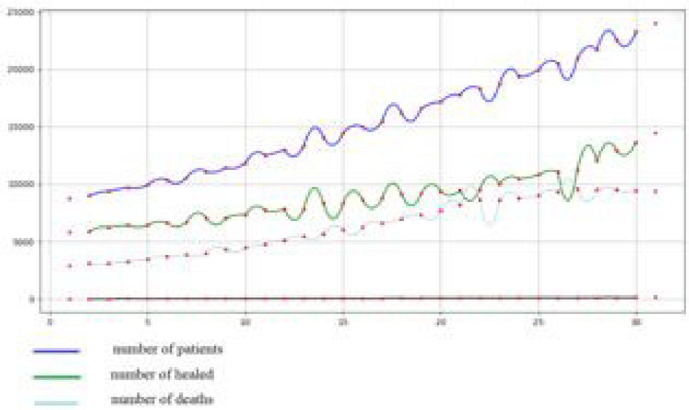
Today, the modeling of natural processes is one of the most pressing issues in applied mathematics. In particular, many results are obtained in the field of signal recovery in various types of geophysical, biomedical, environmental and other fields, forecasting on the basis of spline models built using local interpolation cubic spline function in processing. The application of regular and singular integrals and regular and singular integral equations in approximate calculations on the basis of the considered local inertial spline function is a topical issue in the development of science and technology.

In this thesis, considers the construction of a local interpolation cubic spline function on the basis of accurate data on the number of cases of COVID-19 among the population in the territory of Uzbekistan from 01.07.2020 to 30, and $S_{3(i)}(x)$, $(i = 1 \ldots 7)$ local interpolation cubic spline functions are built, on the basis of which spline models are studied to get good results in predicting the duration of the future pandemic, the number of cases. Areas to reduce these diseases and increase the number of healthy people will be studied.



$S_{3(i)}(x)$, $(i = 1 \dots 7)$, graphs of local interpolation cubic spline functions were checked to reflect each of the data given in the table. This process was analyzed on the basis of numerical processing on the basis of initially obtained experimental products.

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ABOUT A NUMERICAL SOLUTION METHOD OF CONSTANTLY CHANGING STATIONARY SOLUTIONS OF FLOW EQUATIONS IN A CHANNEL NETWORK

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Let us consider a channel network consisting of M channels (branches). It can be either tree type or looped one. Each channel is divided into $N_j (j = 1, 2, ..., M)$ intervals of constant or variable length Axi. The total number of grid points in whole network is SN_j. In addition, let us assume positive direction of flow in each channel corresponding to the direction of increasing of the indexes.

$$\left(h_{i+1} + \frac{\alpha Q_j^2}{2gA_{i+1}^2}\right) - \left(h_i + \frac{\alpha Q_j^2}{2gA_i^2}\right) + \frac{\Delta x_i}{2}\left(\frac{n_M^2 Q_j \left|Q_j\right|}{R_i^{4/3} A_i^2} + \frac{n_M^2 Q_j \left|Q_j\right|}{R_{i+1}^{4/3} A_{i+1}^2}\right) = 0 \tag{1}$$

Similar equations are written for each interval $Axi(i = 1, 2, ..., N_j - 1)$. In this way one obtains a system of $N_j - 1$ algebraic equations with $N_j + 1$ unknowns. There are N_j water levels hi at the nodes and the flow discharge Q_j. The system of equations (1) can be presented in the matrix form:

$$AX = B; \tag{2}$$

where: $A-$ matrix of coefficients, $B = (0, 0, ..., 0)^T$ - vector of right hand side, $X = (h_1, h_2, ..., h_{N_j}, Q_j)^T$- vector of unknowns, $T-$ transposition symbol. When flow in a single channel was considered this system had to be completed by two additional equations resulting from the imposed boundary conditions. However, in the case of channel network such system of equations is written for each branch separately.

To assembly them, apart from the mentioned boundary conditions, which are imposed at pendant nodes only, additional equations must be specified for each junction. Preliminary numerical tests carried out for the considered network showed that the Newton iterative method applied to solve equations (2) failed to converge. This conclusion coincides with the well-established opinion of its poor global convergence properties (Press et al. 1992). On the other hand, application of the modified Picard method described in Subsection 4.3.4 is successful. Therefore the non-linear system of equations (2) is solved as follows:

$$A^* X^{k+1} = B \tag{3}$$

where k is index of iteration and

$$A^* = A(0.5(X^{(k)} + X^{(k-1)})) \tag{4}$$

is modified matrix of coefficients. Numerous tests confirmed that with the presented algorithm it is possible to overcome the problem of poor convergence of the standard Picard method.

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MATHEMATICAL MODELING OF HEAT CONDUCTION PROCESS IN VARIABLE PROPERTY ENVIRONMENT BY NUMBER -ANALYTICAL METHODS

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We consider the equation of heat dissipation in the environment with variable properties.

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) - \vartheta(t)\frac{\partial u}{\partial x},\ 0 < t \le T,\ a < x < b \tag{1}$$

$$u\left(0,x\right) = u_0\left(x\right) \ge 0,\ a \le x \le b \tag{2}$$

$$u\left(t,a\right) = u_1\left(x\right) > 0,\ 0 \le t \le T \tag{3}$$

$$u\left(t,b\right) = u_2\left(x\right) > 0,\ 0 \le t \le T \tag{4}$$

Where $c = const,\ \rho = const,\ k = u^\delta, v = v(t)\ \ and\ \ \delta > 0$
$u = u(t,x) \ge 0$ the solution sought.

Equation (1) represents a series of physical processes: they represent the process of reaction diffusion in a nonlinear environment, the process of heat dissipation in a non-homogeneous nonlinear environment, the filtration of liquids and gases in a nonlinear environment, and they represent the law of polytherapy and other nonlinear processes. We consider the solution for equation (1) as follows.

$$u(t,x) = W(t, \gamma(t,x)) \tag{5}$$

We put (5) in equation (1) and formulate the following equation :

$$c\rho\left[W_t + \gamma_t W_\gamma\right] = \frac{\partial}{\partial x}\left(\gamma_x W^\delta W_\gamma\right) - \vartheta(t)\gamma_x W_\gamma \tag{6}$$

$$\gamma(t,x) = c\rho x - \int_0^t \vartheta(\eta)d\eta \tag{7}$$

We put (7) in equation (1) and formulate the following equation:

$$\frac{\partial}{\partial \gamma}\left(W^\delta \frac{\partial W}{\partial \gamma}\right) - \frac{1}{c\rho}\frac{\partial W}{\partial t} = 0 \tag{8}$$

We look for a self - similar solution for this equation as follows :

$$W_A\left(t, \gamma(t,x)\right) = (T-t)^{\alpha} f(\xi), \xi = \gamma(T-t)^{-\beta} \tag{9}$$

We find the self-similar solution as follows:

$$W_A\left(t, \gamma(t,x)\right) = (T-t)^{\alpha} \left(\frac{\delta\beta}{2c\rho} \left(c_1 + \xi^2\right) \right)^{\frac{1}{\delta}} \tag{10}$$

$Af = \frac{\beta-\alpha}{c\rho} f$, $\forall f \geq 0$, $c, \rho \geq 0$ A self-similar solution has been found

$\beta \geq \alpha = \frac{2\beta-1}{\delta} \Rightarrow \beta \geq -\frac{1}{\delta-2}$ high solution for, if $Af \geq 0$

$Af \leq 0$ if (1) for the lower solution will be, $\beta \leq \alpha = \frac{2\beta-1}{\delta} \rightarrow \beta \leq -\frac{1}{\delta-2}$.

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CONSTRUCTION OF OPTIMAL DIFFERENCE FORMULA

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We know that the solution of many practical problems leads to the solution of differential equations or their systems. Although differential equations have so many applications, only a small number of them can be solved with precision using elementary functions and their combinations. Even in the analytical analysis of differential equations, their application can be inconvenient due to the complexity of the obtained solution. If it is impossible to find an analytical solution to a differential equation, or if it is very difficult to obtain, we can try to find an approximate solution. There are two traditional approaches to finding an approximate solution:

1. Semi-analytical methods. Sometimes the solution to a differential equation is expressed using simpler functions. We can use series, integral equations, or asymptotic methods.

2. Numerical solutions. Discrete numeric values can represent the solution to a differential equation with a certain degree of accuracy. Currently, such a large number of matrices and related tables or graphs are obtained with the help of computers. This provides efficient approximate solutions that were previously not possible.

This paper focuses mainly on second approach.

In the following section $[0, 1]$ we need to find the approximate solution of the differential equation given by the initial condition $y(0) = y_0$

$$y' = f(x, y). \tag{1}$$

We divide this $[0, 1]$ interval into N pieces of the length $h = \frac{1}{N}$ and give the approximate values y_n of the function $y(x)$, $n = 0, 1, \ldots, N$ calculate at nodes points $x_n = nh$. A classic example of such a method is the Euler method. Using this method, the approximate solution of the differential equation is calculated as follows: To find the approximate value of the function x_{n+1} at node y_{n+1}, we use the approximate value of y_n at node x_n: $y_{n+1} = y_n + hy'_n$, where $y'_n = f(x_n, y_n)$, so that y_{n+1} is a linear combination of the values of the given function and its first-order derivative at the node x_n.

In this paper, we considered the problem of construction of optimal difference formulas for approximate solution of the Cauchy problem in the $W_2^{(2,0)}(0, 1)$ space. Here, using the discrete analogue of the differential operator $\frac{d^4}{dx^4} + 2\frac{d^2}{dx^2} + 1$, the optimal difference formula is constructed.

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OPTIMAL QUADRATURE FORMULA FOR NUMERICAL CALCULATION OF FOURIER INTEGRALS

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Numerical calculation of integrals of highly oscillating functions is one of the more important problems of numerical analysis, because such integrals are encountered in applications in many branches of mathematics as well as in other science such as quantum physics, flow mechanics and electromagnetism. Main examples of strongly oscillating integrands are encountered in different transformation, for example, the Fourier transformation and Fourier-Bessel transformation. Standard methods of numerical integration frequently require more computational works and they cannot be successfully applied. The earliest formulas for numerical integration of highly oscillatory functions were given by Filon in 1928.

$$I\left[f;\omega\right]=\int_a^b e^{i\omega x}f(x)dx$$

is based on piecewise approximation of $f(x)$ by arcs of the parabola on the integration interval. Then finite integrals on the subintervals are exactly integrated.

In this paper, the optimal quadrature formula for approximate evaluation of Fourier coefficients $\int_0^1 e^{2\pi i\omega x}\varphi(x)dx$ is constructed for functions of the space $W_2^{(3,0)}(0,1)$. At the same time explicit formulas for optimal coefficients, which are very useful in applications, are obtained. The obtained formula is exact for the exponential-trigonometric functions e^{-x}, $e^{\frac{x}{2}}\cos\left(\frac{\sqrt{3}}{2}x\right)$ and $e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x\right)$. In particular, as consequences of the main result the new optimal quadrature formulas for approximate evaluation of integrals $\int_0^1\cos(2\pi\omega x)\varphi(x)dx$ and $\int_0^1\sin(2\pi\omega x)\varphi(x)dx$ are obtained. Furthermore, the order of convergence of the constructed optimal quadrature formula is studied.

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CONSTRUCTION OF OPTIMAL INTERPOLATION FORMULA EXACT FOR TRIGONOMETRIC FUNCTIONS

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One of the main problems of computational mathematics is the optimization of computational methods in functional spaces. Optimization computational methods are well demonstrated in the problems of the theory of interpolation formulas. The classical method of its solution consists of construction of interpolation polynomials. However polynomials have some drawbacks as an instrument of approximation of functions with specifics and functions with small smoothness.That is why splines are used instead of interpolation polynomials of high degree.There are algebraic and variational approaches of construction in the spline theory. In algebraic approach splines are considered as some smooth piecewise polynomial functions. In the variational approach splines are elements of Hilbert or Banach spaces minimizing certain functionals. Then we study the problems of existence, uniqueness, and convergence of splines and algorithms for constructing them based on their own properties of splines.

In this paper, we study the problem of constructing an optimal interpolation formula exact for trigonometric functions $\sin(x)$ and $\cos(x)$ in the space $W_2^{(3,1)}(0,1)$ by Sobolev method. Here, a system of linear algebraic equations for the coefficients of the optimal interpolation formula is obtained and using the discrete analogue of the operator
$\frac{d^6}{dx^6} + 2\frac{d^4}{dx^4} + \frac{d^2}{dx^2}$, the optimal interpolation formula is constructed.

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EVALUATING THE EFFECTIVENESS OF CLOUD AND VIRTUAL TECHNOLOGIES

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Statement of the problem. The adoption of cloud technology is increasing day by day. Many organizations are deciding to shift their businesses to cloud to benefit from scalability, resilience and cost reduction characteristics. However, due to the size and complexity of cloud environments,failures occur frequently, impacting cloud reliability significantly.

The goal is to design an effective reliability evaluation model for cloud and conduct the integration of evaluation model in some aspects of cloud resource management. More specifically,reliability evaluation will be integrated into the processes of VM allocation,VM migration for multi-cloud environments, and cloud federation formation.

Cloud computing is a computing concept based on parallel computing, distributed computing and grid computing. The main goal of cloud computing model is to make better use of its resources and achieve better service performance in a cost-effective manner.

Cloud computing paradigm also introduces important challenges for Cloud Service Providers(CSPs), such as performance guarantee,resource limitation,disaster-recovery planning, regional distribution of workloads,and legal issues.

Most definitions of reliability are expressed either by including failure-free operation of a system, or a combination of quality attributes like accuracy, fault tolerance, fault recovery and availability. For the definition of reliability, the following one given by the IEEE Reliability Society is an authoritative one:

"Reliability is a design engineering discipline which applies scientific knowledge to assure that a product will perform its intended function for the required duration within a given environment. This includes designing the ability to maintain, test and support the product through its total life cycle"

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THE APPLICATION OF DATA ENVELOPMENT ANALYSIS (DEA) APPROACH FOR SOCIO-ECONOMIC PERFORMANCE

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DEA is a non-parametric method that utilizes linear programming to measure the level of efficiency of comparable decision-making units (DMU) by employing multiple inputs and outputs. The CRS model, named by its developers Charnes, Cooper and Rhodes, is based on fixed or constant returns-to scale. This actually means that the proportional increase of all the inputs results in the same proportional increase of all the outputs. In the VRS model the input increase does not result in proportional changes of the output.

In this paper we consider the level of efficiency of comparable decision-making units (DMU) by employing multiple inputs and outputs.

$$Efficiency = \frac{f(Outputs)}{g(Inputs)}$$

where $f(.), g(.)$-some functions. Different potential socio-economic determinants that were used in these studies are selected based on the research scope and classified into three main categories. The focus of the studies in the first category is mostly on economic indicators. The most frequently used are gross domestic product (GDP), employment and capital stock. The studies in the second category underline the significance of environmental issues, combining economic indicators with ecological, usually the undesirable ones. Most commonly, these are greenhouse gas emissions to the environment, such as carbon dioxide, nitrous oxide, etc. In the studies from the third category, the emphasis is placed on the impact of energy supply and consumption. Therefore, alongside economic and environmental ones, energy indicators, such as gas, power, coal and oil consumption, are usually employed. Although researchers' interest and engagement in formulating and applying analytical and modelling techniques in energy and environmental studies goes much further back in time, it has rapidly intensified over the past $15-20$ years. Consequently, a significant portion of the studies, particularly of more recent ones, tackle environmental and energy issues in performance measurement[2]. CRS and VRS models of DEA were used to evaluate the efficiency of international economic relations.

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ON THE STABILITY OF SOME LOTKA AND VOLTERRA MODELS

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For the first time, V. Volterra [1, 2] and A.J. Lotka [3] were substantiated research of a new form of differential equations called the Lotka-Volterra equations in the most complete form. Currently, the relevance of works on the study of these equations is associated with their wide application in various fields of science and technology including biology, ecology, economics, automatic control, etc. Uzbek scientists have made a great contribution to the development of theoretical and applied research in this area [4, 5, etc.].

In this paper, we consider the stability problem of nonlinear Lotka-Volterra equations that model the interaction process of competing processes. Such models are actively investigated in the problems of the existence of different species in ecology, sustainable competition of forms that produce the same products in economy.

The following Lotka-Volterra system is considered

$$\frac{dx(t)}{dt} = D(x(t))(A + BF(x(t)) + QF(x(t - h(t)))) \tag{1}$$

where $x \in R^n$, $D = (d_1(x_1), d_2(x_2), \ldots, d_n(x_n))^T$, $F = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T$, $d_k, f_k \in C^1(R^+ \to R^+)$, $d'_k > 0$, $f'_k > 0$, $A = (a_1, a_2, \ldots, a_n)^T$, $B, L \in R^{n\times n}$, $h = (h_1(t), h_2(t), \ldots, h_n(t))^T$, $h_k \in C^1$ $(R^+ \to [-h_0, 0])$, $(h_0 > 0)$, $k = 1, 2, \ldots, n$, $(\cdot)^T$ is a transposition operation.

Assume that $D(0) = 0$, and the matrix equation

$$(B + Q)y + A = 0$$

has the unique solution $y = y^{(0)} = (y_1^{(0)}, y_2^{(0)}, \ldots, y_n^{(0)})^T$, $y_k^{(0)} > 0$, $k = 1, 2, \ldots, n$.

Thus, the system (1) has a finite set of equilibrium positions or the set of equilibrium numbers of populations. In the work, the stability conditions for these states of the system (1) are found on the basis of [5].

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ASYMPTOTICS FOR THE EIGENVALUE OF THE DISCRETE SCHRÖDINGER OPERATOR ON THE TWO-DIMENSIONAL LATTICE

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In this work we look at the discrete Schrödinger operator and find the asymptotics of eigenvalue as $\mu \to \infty$ The Schrödinger operator H_μ is of the form defined by

$$H_\mu = H_0 - V.$$

H_0 is multiplication operator by the function $\varepsilon(\cdot)$:

$$(H_0 f)(p) = \varepsilon(p) f(p), \quad f \in L^2(\mathbb{T}^2), \quad p \in \mathbb{T}^2,$$

where

$$\varepsilon(p) = \sum_{j=1}^{2} (1 - \cos p_j) + \beta(1 - \cos 2p_j), \quad p \in \mathbb{T}^2, \quad \beta \geq 0. \tag{1}$$

The potensial operator is transformed into a rank one integral operator

$$(Vf)(p) = \frac{\mu}{2\pi} \int_{\mathbb{T}^2} f(q) dq, \quad f \in L^2(\mathbb{T}^2). \tag{2}$$

H_0 is a multiplication operator by a function, the perturbation V is the one-dimensional operator and, therefore, in accordance to the Weyl theorem on the stability of the essential spectrum the equality $\sigma_{ess}(H_\mu) = \sigma_{ess}(H_0)$ holds, and the essential spectrum of the operator H_μ consists of the following segment on the real axis:

$$\sigma_{ess}(H_\mu) = [\varepsilon_{\min}, \varepsilon_{\max}],$$

where

$$\varepsilon_{\min} = 0, \quad \varepsilon_{\max} = \begin{cases} 2, & 0 \leq \beta \leq \frac{1}{4}, \\ \frac{(1+4\beta)^2}{4\beta}, & \beta > \frac{1}{4}. \end{cases}$$

For any $\mu \in \mathbb{R}$, we define the Fredholm determinant associated with the operator H_μ as a regular function in $z \in \mathbb{R} \setminus [\varepsilon_{\min}, \varepsilon_{\max}]$ as

$$D(\mu, z) = 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}^2} \frac{dp}{\varepsilon(p) - z} \tag{3}$$

Theorem. $z(\mu)$ satisfies the asymptotics

$$z(\mu) = -\mu + 2(1+\beta) - \left(1+\beta^2\right) \frac{1}{\mu} + O(\frac{1}{\mu^2})$$

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REGULARIZATION OF THE BOUNDARY VALUE PROBLEM FOR THE PARABOLIC EQUATION WITH THE DEGENERATION LINE

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This work is devoted to the construction of the regularized solution of the ill-posed initial-boundary value problem for the inhomogeneous parabolic equation with degeneration line.

Let $Q = \{(x, y, t) : (x, y) \in \Omega,\ 0 < t < T,\ T < \infty\}$, where $\Omega = \{-\pi < x < \pi,\ 0 < y < \pi\}$.

In the region $Q \cap \{x \neq 0\}$, we consider the differential equation

$$u_t + sgn(x)u_{xx} + u_{yy} = f(x, y, t), \tag{1}$$

where $f(x, y, t)$ given function of the source.

Formulation of the problem. Find a function $u(x, y, t)$ satisfying equation (1) in the region $Q \cap \{x \neq 0\}$ and the following conditions:
initial

$$u|_{t=0} = \varphi(x, y),\ (x, y) \in \bar{\Omega}, \tag{2}$$

boundary

$$\begin{aligned} u(-\pi, y, t) = u(\pi, y, t) = 0,\ 0 \le y \le \pi,\ 0 \le t \le T, \\ u(x, 0, t) = u(x, \pi, t) = 0,\ -\pi \le x \le \pi,\ 0 \le t \le T, \end{aligned} \tag{3}$$

and gluing conditions

$$\left.\begin{aligned} u(-0, y, t) = u(+0, y, t), \\ u_x(-0, y, t) = u_x(+0, y, t), \end{aligned}\right\},\ 0 \le y \le \pi,\ 0 \le t \le T, \tag{4}$$

where $\varphi(x, y)$ is sufficient smooth function and satisfies the consistency conditions.

In this paper, approximate solutions of the problem (1) - (3) are constructed by regularization and quasi-inversion methods. In the corresponding functional space, an estimate is obtained for the norm of the difference between the exact and approximate solutions corresponding to the approximate data. A formula for calculating the regularization parameters by minimizing the resulting estimate is given.

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THE FINAL SPEED OF PROPAGATION OF PERTURBATIONS AND LOCALIZATION IN THE CASE OF STRONG ABSORPTION OR SOURCE

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In this paper we study the numerical analysis of solutions of a nonlinear filtration problem. A self-similar equation was constructed. For difference schemes, we used a two-layer difference scheme. Results of numerical experiments showed that the results agreed with physics of a nonlinear filtration.

Consider the Cauchy problem in the field of $Q = [0,\infty) \times R^N$ Cauchy problem

$$\frac{\partial u}{\partial t} = \nabla \left(|x|^n u^{m-1} \left|\nabla \mathrm{u}^{\mathrm{k}}\right|^{p-2} \nabla \mathrm{u} \right) \pm u^{\beta} \tag{1}$$

$|x|^n u^{m-1} |\nabla u^k|^{p-2} \nabla u = 0$ on the boundary Γ (1)

$$|x|^n\, u^{m-1} \left|\nabla \mathrm{u}^{\mathrm{k}}\right|^{p-2} \nabla\, \mathrm{u}|_{x \to +\infty} \tag{2}$$

This task describes the processes of nonlinear filtering diffusion heat conduction, when the thermal conductivity is a power function of the derivative of the presence of absorption.

Investigation of various properties of solutions of the problem when $\beta > 1$ is dedicated to a great number of works in which the issues of global solvability (Fujita, Samara, etc.), evaluation of solutions and fronts (Kalashnikov, Kershner, etc.)

Under some suitable assumptions, the existence, uniqueness and regularity of a weak solution to the Cauchy problem (1)-(2) and their variants have been extensively investigated by many authors (see [1–3] and the references therein).

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INFORMATION SECURITY PROBLEMS IN THE MODERN GLOBALIZED WORLD

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In this article the mechanism of the legal adjusting of informative safety is analysed in the modern globalized world, because on conditions of mushroom growth of global informative society, deployment of informatively-communication technologies this problem acquires in all spheres of life of the special value. An aim of the article is exposition of basic results of research in relation to forming of the system of knowledge to the decision of essence and maintenance of problem of informative safety in the modern globalized world. Going out theoretical approaches and practice of legal relationships, that found a reflection in publications, home legislation, and also comparison of home practice with foreign experience, legal relationships in the field of informative safety are examined within the framework of the field of administrative law [5].

Ways to improve the efficiency of administrative and legal regulation of information security are analyzed, taking into account the intersectoral nature of administrative legal relations in the field of information security in the context of the need to bring the national legislation over of the Republic of Uzbekistan to the international standards according to the declared foreign-policy priorities [1]. Importance of development of global informative society and use of new possibilities are grounded for the decision of his problems [2-4].

Information security is achieved through a balance between information rights and freedoms of various subjects of law and the protection of national information sovereignty. The issue of information security and national security in general is primarily a question of balance between human rights and interests, and the competence and interests of state power, a balance that can be established only with the help of legal norms.

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ON SELF-SIMILAR SOLUTIONS OF EQUATIONS - NONSTATIONARY FILTRATION IN TWO COMPONENT NONLINEAR MEDIA

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In this paper, we construct asymptotic representations of self-similar solutions to system (1) near the front, necessary and sufficient signs of their existence have been found. A new way is proposed obtaining self-similar (invariant-group) equations based on the splitting of the original system (1).

Consider in the domain Q=$\{(t,x)$: 0< t < ∞, x$\in$R$^N\}$ a parabolic system of two quasilinear equations

$$\frac{\partial v_i}{\partial t} = \nabla(v_i^{\sigma_i}\nabla v_i) + \varepsilon v_{3-i}^{q_i}(i = 1, 2) \tag{1}$$

where $\epsilon=\pm 1$, σ_i, q_i $(i=1,2)$ - are positive real numbers, $\nabla(.)\text{-}grad_x(.)$, $v_i = v_i(t,x) \geq 0$ $(i=1,2)$ are the desired solutions.

System (1) describes filtration processes in two-component nonlinear media in the presence of infiltration (source or sink) whose power depends on the components of the equations in a power manner. System (1) also describes other processes [1,2]. The global solvability of the Cauchy problem and the first boundary value problem for system (1) were considered in [1,2]. However, the question of the existence and asymptotic behavior of solutions remained open [1, 2].

In this paper, we construct asymptotic representations of self-similar solutions to system (1) near the front, necessary and sufficient signs of their existence have been found. A new way is proposed obtaining self-similar (invariant-group) equations based on the splitting of the original system (1) [3,4].

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ESTIMATION OF THE SOLUTION OF THE CAUCHY PROBLEM FOR THE EQUATION OF HEAT CONDUCTIVITY WITH NONLINEAR ABSORPTION

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In this paper, we investigate the global solvability, estimates for solutions from below and from above of one heat equation with nonlinear absorption. The behavior of the free boundary is established, and invariant solutions are found. The condition for global solvability and asymptotic behavior of solutions are obtained. Based on these results, numerical experiments are performed.

Consider in $Q = \{(t,x) : t > 0, x \in R^N\}$ the Cauchy problem

$$Lu = -\frac{\partial u}{\partial t} + \Delta u - k(t,x)f(u,x) = 0, \tag{1}$$

$$u|_{t=0} = u_0(x) \geq 0, \ \ x \in R^N \tag{2}$$

Problem (1), (2) describes various processes of heat conduction, diffusion, filtration, etc. in the presence of absorption, the power of which is equal to

$$k(t,x)\ f(u,x).$$

A particular case of problem (1), (2), when $k(t,x) = 1, \ \ f(u,x) = u^{\beta}$ is devoted to work [1], where the properties of the solution to this problem are comprehensively investigated by analyzing the properties of self-similar solutions and it is shown that the nature, asymptotics of the solution is very complex, due to the nonlinear member. It was established that the properties of the solution depend on the value of the parameter β, the dimension of the spaces, and the smoothness $u_0(x)$.

In this paper, we study the global solvability of problem (1), (2). Conditions for the localization of the solution and upper and lower estimates for the solution of problem (1), (2) are obtained. A method for constructing the upper and lower solutions is proposed.

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MATHEMATICAL MODELS FOR FLOOD AND CHANNEL FLOWS

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The general mathematical model governing the flow of a fluid in many applications, in which the continuum hypothesis is valid, is the Navier-Stokes equations (NSE). Such applications include flows in pipes, seas, oceans, around aircraft and flooding. For a given application, the NSE are complimented with boundary and initial conditions, which in principle, should enable to completely solve the equations for the problem under investigation. However, for free-surface flows such as flood and channel flows, amongst many others, the position of the boundary of the flow domain is usually not known a priori. This makes a direct application of the NSE to free-surface flow problems diffcult as the boundary conditions can not be directly specified. In addition to the above diffculty, solving the full 3D NSE is computationally expensive as there does not exist an exact analytical solution for the general 3D NSE. Therefore, to solve a free-surface flow problem, further model derivation is usually carried out, with the hope of simplifying the NSE into less computationally expensive models and also to circumvent the unknown boundary position problem.

For some flows, like those in seas, rivers, floods, atmosphere, and open channels, amongst many others, the horizontal length scales (like river length) are much larger than the vertical length scales, like fluid depth. This allows to assume that vertical component of acceleration is negligible, which in turn, leads to the assumption of hydrostatic pressure distribution, which means that the net pressure exerted on a fluid particle is only due to the force exerted on it by other fluid particles lying above it.

With the above approximation, the shallow water theory is established where the unknown free-surface position, $\eta(X^{\rightharpoonup}, t)$ is formulated as part of the solution of the problem.

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SOLUTION OF LOGISTICS PROBLEMS ON THE BASIS OF INTELLECTUAL ANALYSIS METHODS

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The study of the analysis of logistics activities has shown that scientific and practical publications to varying degrees consider the methods used in the study of this topic. Despite certain steps in this direction, a universal and generally accepted assessment methodology is not recognized by most researchers. The methodology of the research of this activity is characterized by multidirectional and a wide range of research subjects, established methodological foundations, natural principles, diverse approaches and aspects under consideration.

It is advisable to classify methods for assessing logistics activities on the basis of individual logistics target functions, logistics tasks of different classes, as well as a set of logistics subsystems. To do this, the target functions of logistics should include compliance with the terms of cargo delivery, minimizing costs in supply chains, pricing of logistics services, management of logistics personnel. Logistics subsystems are distinguished as one of the basic classification features of methods used in logistics methodology; on the basis that each of its subsystems has its own specific features, which consist in the difference in functional purpose, applied technical devices, procedures and operations. The implementation of target functions in subsystems has ambiguous approaches and different methods, despite the presence of the same traffic objects and a common target function for the timely delivery of goods [1,2].

During research, software is currently being developed for logistics firms to automate their business process. As developing stack choosen php / laravel, html, css, js / vue.js. In software will be roles: adminstrator, business owner, manager and dispatcher. Software will predict based on accumulated data using data mining methods. After collecting new data, new prediction model will be regenerated.

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MOMENT INEQUALITIES FOR DEPENDENT RANDOM FIELDS WITH VALUES IN c_0 SPACE

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Let $\xi(t), t \in Z^2$ be a real valued random field of independent identically distributed random variables.We consider random field $\{X(t), t \in Z^2\}$ with values in c_0 (a space of all sequences $x = (x^{(1)}, x^{(2)}, ...)$ such that $\lim\limits_{n\to\infty} x^n = 0$ with a norm $\|x\| = \sup\limits_i |x^{(i)}|$). We denote by $\{e_i, i \geq 1\}$ a standart basis of c_0, (i.e. $e_i = (0, 0, ..., 1, 0...)$ where i-th component is 1 and others are 0)and $X(t) = \sum\limits_{i=1}^{\infty} X^{(i)}(t)e_i$.

We assume that the following representations hold

$$X^{(i)}(t) = g_i\left(\xi(t-s), s \in Z^2\right), \quad i = 1, 2, ... \tag{1}$$

where g_i is a measurable function $i = 1, 2, ...$

Define

$$\overline{X}^{(i)}(t) = g_i\left(\xi^*(t-s), s \in Z^2\right)$$

where

$$\xi^*(j) = \begin{cases} \xi(j), & if \;\; j \neq 0, \\ \xi^{|}(0), & if \;\; j = 0. \end{cases}$$

and $\xi^{|}(t),\ t \in Z^2$ is an independent copy of $\xi(t),\ t \in Z^2$.

Denote

$$\delta_i(t,p) = \left(E\left|X^{(i)}(t) - \overline{X}^{(i)}(t)\right|^p\right)^{1/p}$$

$$\Delta_i(p) = \sum_{t\in Z^2} \delta_i(t,p).$$

We establish moment inequalities for

$$\mathrm{S}_\Gamma = \sum_{t\in\Gamma} X(t)$$

where Γ is a finite subset of Z^2. One of the results is the following.

Theorem. Let $\{X(t), t \in Z^2\}$ be a random field with values in c_0 satisfying (1). Assume that the following conditions hold

$$EX(t) = 0,\ E\|X(t)\|^p < \infty, \quad p \geq 2$$

Then there exists a constant $C > 0$ such that

$$E\|S_\Gamma\|^p \leq C|\Gamma|^{p/2} \sum_{i=1}^{\infty} \Delta_i^p(p)$$

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DATA FITTING PROBLEMS UNDER INTERVAL UNCERTAINTY

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We consider a linear regression model

$$y = \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_m,$$

where the unknown coefficients β_1, β_2, ..., β_m should be determined from measured values of the independent variables x_1, x_2, ..., x_m, and dependent variable y. The measurements are subject to inaccuracies and uncertainties which are assumed to have interval form. An interval $n \times m$-matrix $\boldsymbol{X} = (\boldsymbol{x}_{ij})$ and an interval n-vector $\boldsymbol{y} = (\boldsymbol{y}_i)$ represent, respectively, the input data and output responses of the model, such that $x_1 \in \boldsymbol{x}_{i1}$, $x_2 \in \boldsymbol{x}_{i2}$, ..., $x_m \in \boldsymbol{x}_{im}$, $y \in \boldsymbol{y}_i$ in the i-th measurement, $i = 1, 2, \ldots, n$. We have to find the coefficients β_j, $j = 1, 2, \ldots, m$, that best fit the above linear relation for the interval data given.

A family of values of the parameters β_j, $j = 1, 2, \ldots, m$, will be called *compatible* with the interval data $(\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2}, \ldots, \boldsymbol{x}_{im})$, $\boldsymbol{y}_i$, $i = 1, 2, \ldots, n$, if, for every index i, there exist such $x_{i1} \in \boldsymbol{x}_{i1}$, $x_{i2} \in \boldsymbol{x}_{i2}$, ..., $x_{im} \in \boldsymbol{x}_{im}$, and $y_i \in \boldsymbol{y}_i$ that $\beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_m x_{im} = y_i$.

The set of all the parameters compatible with the data form a *information set* for the problem. It is the so-called united solution set $\Xi_{uni}(\boldsymbol{X}, \boldsymbol{y})$ to the interval system of linear equations $\boldsymbol{X}\beta = \boldsymbol{y}$, defined as

$$\Xi_{uni}(\boldsymbol{X}, \boldsymbol{y}) = \{\, \beta \in \mathbb{R}^m \mid (\exists X \in \boldsymbol{X})(\exists y \in \boldsymbol{y})(X\beta = y) \,\}.$$

However, in some practical cases the above definition turns out to be inadequate.

Let us consider the situation when measuring the output is performed *after* the input values are fixed. If we get an interval $\boldsymbol{y}_i$ as the result of the ith output measurement, then it is natural to expect that the actual value $\tilde{y}$ on output belongs to $\boldsymbol{y}_i$ no matter what are the input values x_1, x_2, ..., x_m within their respective intervals $\boldsymbol{x}_{i1}$, $\boldsymbol{x}_{i2}$, ..., $\boldsymbol{x}_{im}$. However, the above definition of the compatibility allows such coefficient families β_1, β_2, ..., β_m that the function value $y = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_m x_{im}$ may not lie within $\boldsymbol{y}_i$ for some $x_{i1} \in \boldsymbol{x}_{i1}$, $x_{i2} \in \boldsymbol{x}_{i2}$, ..., $x_{im} \in \boldsymbol{x}_{im}$.

A family of values of the parameters β_j, $j = 1, 2, \ldots, m$, is called *strongly compatible* with the interval data $(\boldsymbol{x}_{i1}, \boldsymbol{x}_{i2}, \ldots, \boldsymbol{x}_{in})$, $\boldsymbol{y}_i$, $i = 1, 2, \ldots, n$, if, for every index i and for each point representatives $x_{i1} \in \boldsymbol{x}_{i1}$, $x_{i2} \in \boldsymbol{x}_{i2}$, ..., $x_{im} \in \boldsymbol{x}_{im}$ there exists such $y_i \in \boldsymbol{y}_i$ that $\beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_m a_{im} = y_i$.

The information set that satisfies the definition of strong compatibility is the tolerable solution set $\Xi_{tol}(\boldsymbol{X}, \boldsymbol{y})$ to the interval system of linear equations $\boldsymbol{X}\beta = \boldsymbol{y}$, defined as

$$\begin{aligned} \Xi_{tol}(\boldsymbol{X}, \boldsymbol{y}) &= \{\, \beta \in \mathbb{R}^m \mid (\forall X \in \boldsymbol{X})(\exists y \in \boldsymbol{y})(X\beta = y) \,\} \\ &= \{\, \beta \in \mathbb{R}^m \mid (\forall X \in \boldsymbol{X})(X\beta \in \boldsymbol{y}) \,\}. \end{aligned}$$

In general, there holds $\Xi_{tol}(\boldsymbol{X}, \boldsymbol{y}) \subseteq \Xi_{uni}(\boldsymbol{X}, \boldsymbol{y})$.

In our work, we develop a practical technique for computing strong compatibility estimates for the parameters of linear regressions, discuss their interpretation and features. We show that the strong compatibility estimates are much more adequate in many practical situations, being easilly computable at the same time.

CREATING AND USING ELECTRONIC TEXTBOOKS WITH MULTIMEDIA SOFTWARE

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The use of new pedagogical technologies in the educational process, including computer hardware and software, allows students to significantly increase the process of mastering. This is determined by the quality and weight of the materials used.

Today, a new generation of textbooks - e-textbooks and their use in education - is being put into practice. E-textbooks (EDs) have several advantages over conventional editions. First of all, it is very convenient to update and disseminate information. Second, the search for the required information in a large amount of data takes a very short period of time. Today, several types of them are used in the development and implementation of e-textbooks. The most common type is web page viewing.

Multimedia is a set of software and hardware that communicates with a computer using a variety of environments. Multimedia is a great way to illustrate an event. The use of multimedia tools allows you to actively use sound, animation, multimedia, color, graphics.

It consists of the organization of lessons in an interactive way on the basis of modern information and pedagogical technologies through the use of multimedia tools, including animation and video materials in the educational process. Multimedia tools cover the entire course or parts of it that are difficult to master. Using these techniques, students can manage and organize effective learning activities.

Possibilities of animated information to increase the efficiency of the educational process:

– Virtual access, openness of students to future specialties, research laboratories;

–Describe processes, events, scenes, and their various models that are difficult to understand (rarely performed, continuous);

–To feel and understand the progress of a problematic process by controlling its performance in a live way;

–Demonstration relies on attention-grabbing, impression-sensitivity, structural-logical methods of influence.

Based on the above requirements, an ED called "Animation Creating Softwares" was created for IT majors in higher education. This ED illustrates the capabilities of animation and multimedia software. Designed for ordinary computer users, this Animation Creator course covers the capabilities of MS PoverPoint, Adobe Animate, Advanced GIF Animator, Ulead Video Studio, Babarosa Gif Animator, eZ-Moniton, MoHo, KoolMoves, and JavaScript.

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INVERSION OF FRACTIONAL INTEGRAL OF $\psi-$ RIEMAN-LIOUVILLE TYPE

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In [1], a generalized form for fractional Riemann-Liouville operators, called fractional operators of the Riemann-Liouville type, is presented, important properties of new generalized ψ-Riemann-Liouville operators in the space $L^p([a, b])$ are obtained and proved.

Fractional derivatives of ψ–Marchaud type have not yet been studied. In this article, we introduce new generalizations of fractional derivatives of ψ–Marchaud type on an interval. Theorems of inversion and description of fractional integrals of the ψ–Riemann-Liouville type of functions in the space $L^p([a, b])$ are proved.

In [1, 2], fractional integrals and derivatives of order α $(\alpha > 0)$ on an interval $[a, b]$ $(-\infty \le a < b \le \infty)$, determined for $x > a$ are considered by the following equations:

$$\left(I_{a+}^{\alpha,\psi}\varphi\right)(x) = \frac{1}{\Gamma(\alpha)}\int\limits_a^x \psi'(t)\left(\psi(x) - \psi(t)\right)^{\alpha-1}\varphi(t)\,dt,$$

$$\left(D_{a+}^{\alpha,\psi}f\right)(x)=\left(\frac{1}{\psi'(x)}\frac{d}{dx}\right)^{n}I_{a+}^{n-\alpha,\psi}f(x),$$

where $n=[\alpha]+1$, $[\alpha]$ means an integral part of α, $\psi\in C^1([a,b])$ is a positive increasing function such that $\psi'(x)\neq 0$ for all $x\in(a,b)$. For $0<\alpha<1,\ 0<\varepsilon<1$, we assume that:

$$\left(D_{a+,\varepsilon}^{\alpha,\psi}f\right)(x)=\frac{f(x)}{\Gamma(1-\alpha)(\psi(x)-\psi(a))^{\alpha}}+\frac{\alpha}{\Gamma(1-\alpha)}\int_{a}^{\psi^{-1}(\psi(x)-\varepsilon)}\frac{f(x)-f(t)}{(\psi(x)-\psi(t))^{\alpha+1}}\,\psi'(t)\,dt$$

which we call the "truncated" fractional derivative of ψ- Marshaud type.

Theorem 1. Let $f(x)=\left(I_{a+}^{\alpha,\psi}\varphi\right)(x)$, $\varphi\in L^p([a,b])$, $0<\alpha<1$, $1\leq p<\infty$, $0<\varepsilon<1$ and let $\psi\in C^1([a,b])$ be a positive increasing function such that $\psi'(x)\neq 0$ for all $x\in(a,b)$. Then

$$\left(D_{a+}^{\alpha,\psi}f\right)(x)=\lim_{\varepsilon\to 0}\left(D_{a+,\varepsilon}^{\alpha,\psi}f\right)(x)=\varphi(x)$$

where the limit is understood in the space $L^p([a,b])$.

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PUBLIC KEY INFRASTRUCTURE AND ITS FUNCTIONS

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Public Key Infrastructure (PKI) is a set of services for managing keys and digital certificates of users, programs and systems[1]. PKI uses public key technology to: identification of participants in electronic exchange (users, programs, systems);ensuring the confidentiality of information; control over the integrity of information; establishing the origin of information.

Basic PKI components:

- The certification authority, also known as the certification authority (CA), is the main structure that generates digital certificates for subordinate certification authorities and end users. CA is the main component of PKI: it is a trusted third party; this is the server that manages the lifecycle of certificates (but not their actual use).
- A public key certificate (most often just a certificate) is a user/server data and his public key, sealed with an electronic signature of a certification authority. By issuing a public key certificate, the certification authority thereby confirms that the person named in the certificate owns the private key that corresponds to this public key.
- Repository - a repository containing certificates and lists of revoked certificates and serving to distribute these objects to users. In the Resolution of the Cabinet of Ministers of the Republic of Uzbekistan No. 348 "Administrative Regulations for the provision of public services for the registration of an electronic digital signature key and the issuance of an electronic digital signature key certificate through public service centers or official information resources online (remotely)"[2].
- Archive of certificates - the repository of all certificates ever issued (including certificates that have expired). The archive is used to verify the authenticity of the electronic signature, which was used to certify the documents.
- The Request Center is an optional component of the system where end users can request or revoke a certificate.

- End Users are users, applications, or systems who own the certificate and use the public key infrastructure.

The main functions of the certification authority: verification of the identity of future users of certificates; issuance of certificates to users; revocation of certificates; maintaining and publishing certificate revocation lists (CRL) that are used by public key infrastructure clients when they decide to trust a certificate.

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